1 (PhD) PA's as MA's (Yngve)

Model a partial algebra A not as a multialgebra $\hat{A}$, where for $f : S \to T$, $f^{\hat{A}}(x) = \emptyset \Leftrightarrow x \notin dom(f^A)$ but by one where $f^{\hat{A}}(x) = T_{\hat{A}} \Leftrightarrow x \notin dom(f^A)$, i.e., partiality means complete unpredictability.

2 (PhD) ND for behavioral abstraction (Yngve ?)

See - Jo's application to NFR.

3 (PhD) Action refinement via nondeterminism

Events are actual events; their descriptions use only abstract labels.
Idea: $A1(a) \prec b$ is $A1(a) \subseteq b$ and reads as "after a (i.e., $A1(a)$) a b must follow".

sorts:	E(vents)	
opers:	a, b, c, $\ldots \to$ E	(event labels)
	A1: E $\to$ E	(after)
	A2: E $\times$ E $\to$ E	(after 2 events)
	...	

and axioms using ND-logic

$$\begin{array}{rl} x \prec a \ \vdash & A1(x) \prec b, A1(x) \prec c \\ \vdash & A1(a) \prec \sqcup.\{b, c\} \\ \vdash & A1(a) \prec b, A1(a) \prec c \\ A1(a) \prec c \ \vdash & \end{array}$$

i.e.,

after any a, there follows either b or c
eithere after all d follws b or after all d follows c
c never follows after a

0.1 (h.fg.) Algebraic Basics

1. define language,
2. define dual semantics: multialgebra & process (is "after" 'immediate' after or 'eventually' after? – multialgebraically, it is rather 'eventually' – hence 1. below might work.)
3. add parallell composition –
4. see how it relates to ACP, PA, ...

0.2 ...and then

1. Will the refinement for multialgebras give a decent action refinement notion?
2. Is "after" 'immediate' after or 'eventually' after? (multialgebraically, it is rather 'eventually' – hence 1. can work.)
3. Can we make it sufficiently expressive/relevant ?

Can try Russell's theory of moments-as-events, or its refinements...

4 (h.fg) Cats of multialgs

Properties of the categories of mults. with remaining compositional homs (from the theorem).

5 (???) Algebras for Multialgebras

Two ways of obtaining "nondeterministic" power-set algebras from algebras. Given an $A \in Alg(\Sigma)$:

1. extend it to an algebra PA over $\wp(A)$ by:
   (a) $c^{PA} \stackrel{\text{def}}{=} \{c^A\}$
   (b) $f^{PA}(X_1, \ldots, X_n) \stackrel{\text{def}}{=} \bigcup\{f^A(x_1, \ldots, x_n) : x_i \in X_i\}$.

   Here, we get that for all ground terms $t \in \mathcal{GT}_\Sigma : |t| = 1$.
2. make a multialgebra by taking a quotient by an equivalence instead of a congruence. Here, constants remain deterministic, but this may be perhaps treated somehow by introducing new elements into the carrier, possibly, modelling constants in the multialgebra by operations taking "hidden" arguments.

Try to, for one or both of these possibilities,

- determine one/class of algebras for a given multialgebra
  in 2., is any multialgebra $M \rightleftharpoons A/_\sim$ for some algebra A and *equivalence* $\sim$?
- see under what conditions this makes sense: is it always possible? does it always follow a natural (not too arbitrary) construction? if not, then when?

- see what implications this may have if we then transfer other notions (homomorphisms...) and results (initiality...) from algebras to multialgebras
  in 1., this is the question (Brink, etc.) of preservation of properties under transition to power structures – can it be expressed in terms of 1-element identity?
- say if it is reasonable to consider a category of (multi)algebras with quotients by equivalences instead of congruences: the notion of homomorphism, ...?

6 (h.fg) Implementation via morphism SPEC → IMPL (Bjørn Tore)

Instead of abstraction function, try to define implementation I of a specification SP by a signature morphism $\iota : \Sigma_{SP} \to \mathcal{T}_{\Sigma_{IM}}$.

7 (h.fg) Implement a generic proof-checker (Adugna)

User definable:

1. term and formulae syntax (?),
2. rules over 1.

and proof-syntax (?)

8 (???) ND for implementation

cf. Peter Mosses

9 (???) Hyperidentities to model ND

10 (???) ND for graph part of CA

11 (PhD) General relational algebra

Extend, as far as possible, Jónsson-Tarski's relational algebra to relations of arbitrary arities – in particular, what is/should be the meaning of relational composition, complement, inverse, etc. in this generalized context.

See how much of appliations of relations could be captured in this way?

1. relational structures – Boolean composition of relations
2. relational databases – joins, projections etc.
3. (hyper)graphs as relations [**?**, **?**] – graph operations.