



# Observing Distributed Computation A Dynamic-Epistemic Approach

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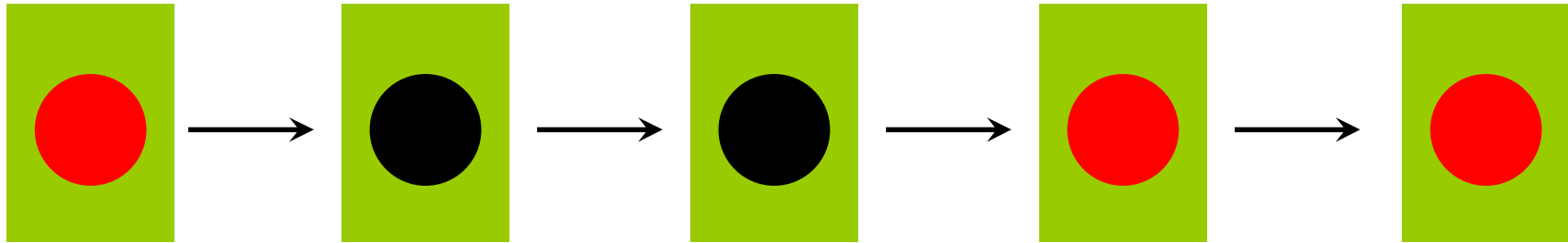
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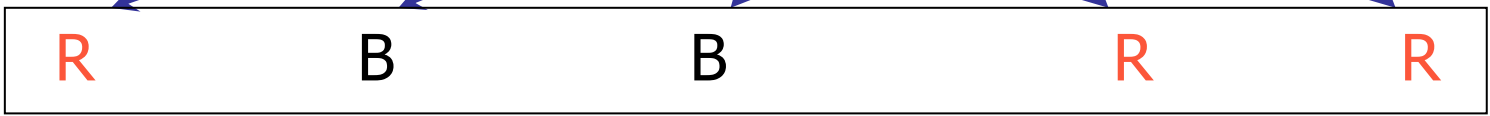
# Observing computational systems



Computational System



the result of computation



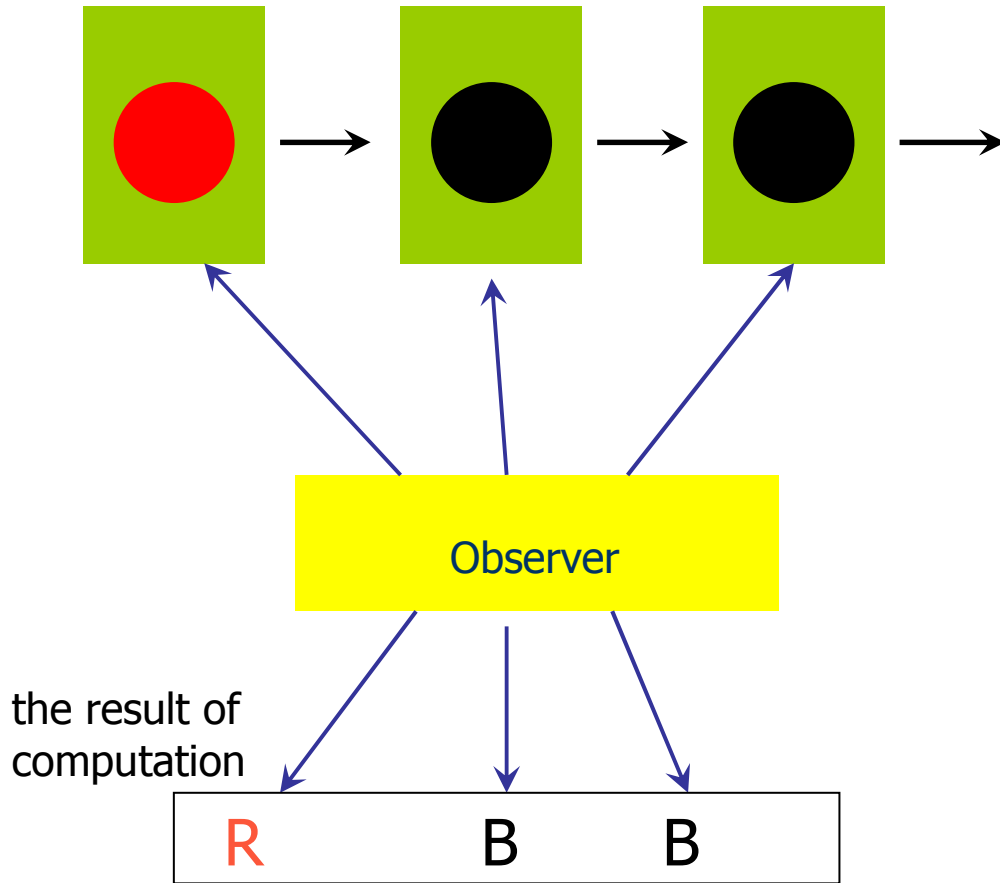
The "result" of a computation is the observation (interpretation) of a process



# Observing computational systems



Computational System



**REG**

**CF**

**Splicing  
systems**

+

+

+

**REG**

**REG**

**REG**

=

=

=

**REG**

**RE**

**RE**

M. Cavaliere, P. Leupold, TCS 2004

M. Cavaliere, N. Jonoska, P. Leupold, Natural Computing 2007



## The “lesson” of the complexity results



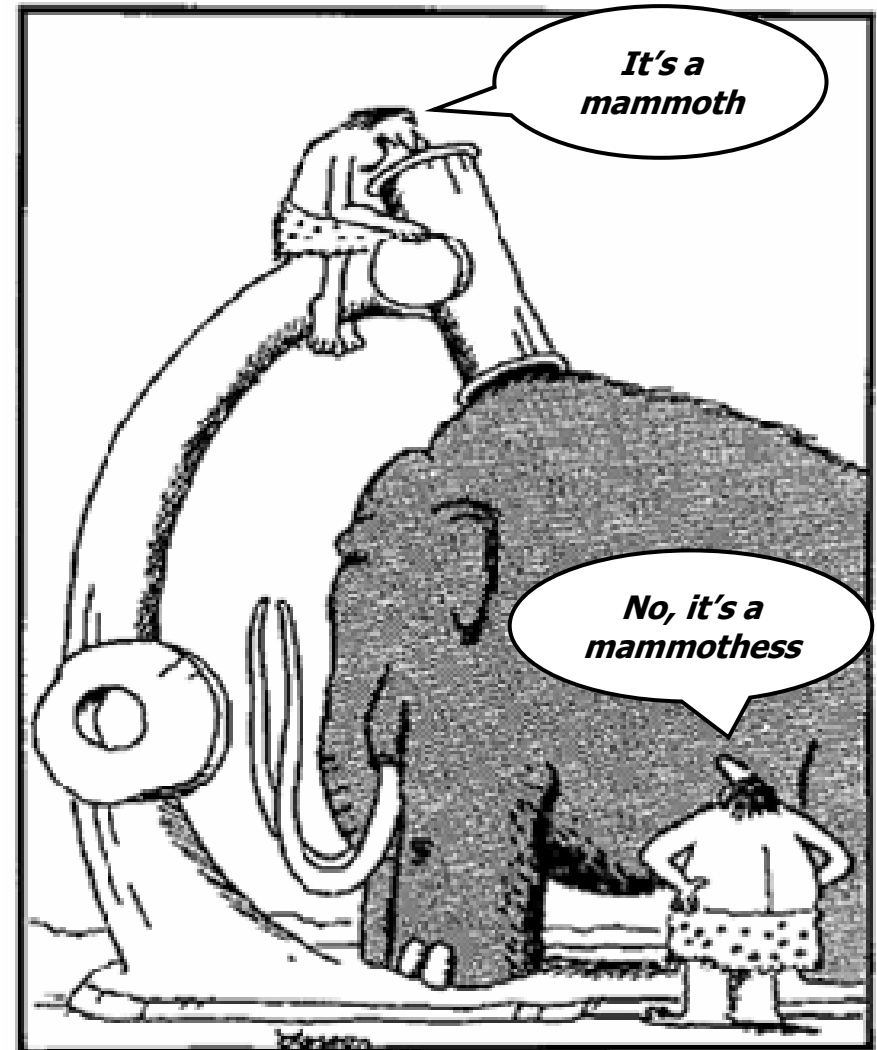
A **computing device** can be seen as composed by two parts:

- An (observed) **dynamic system** that simply "lives", passing from a state to the next one.
- An **observer** (or more), placed outside, that watches and interpret the **evolution** of the system (parts of the system).

The observer uses the trace of the (observed) computational process as a computational resource; thus he can obtain **every** possible computational device without changing the observed system but only by finding the "correct" external **observer**....

*"The real voyage of discovery consists not in seeking new landscapes but in having new eyes"*

(Marcel Proust)



## The “agents” in Computer Science



- **MultiAgent Systems:** are composed of **agents**

- => an agent is a spatially localised and independently observable unit of behavior and computation (e.g. programs or processors running in parallel)

- “An agent is a computational system capable of autonomous action on behalf of its user or owner”* (Michael Wooldridge, *MultiAgent Systems*)

- => is projecting the agent on the observed system

- => no storing/processing information, no knowledge

- **Concurrency:** an **external observer** is a witness of a process

- => for an observer, a process is a black box (observational power)

- => but two distinct observers witnessing the same process have same observational capabilities

- => the observers: no knowledge, no memory, no processing capabilities





- **Epistemic/Doxastic Logics:** multimodal logics

=> formalize the notions of **knowledge** or **belief** possessed by an agent or a group of agents

$K_A\phi$  “agent  $A$  knows  $\phi$ ”

$K_{A_1}K_{A_2}\phi$  “agent  $A_1$  knows that agent  $A_2$  knows  $\phi$ ”

=> semantically based on Kripke structures

=> **Dynamic-Epistemic Logics** combine the epistemic operators with dynamic operators => more information about the observed system

=> **Logics for Epistemic Programs** (Baltag, Moss, Solecki) plays with Kripke structures on different levels and with their “products”



## An alternative approach



- A multimodal logic – **decidable** and **completely axiomatized** (Hilbert style)

⇒ For a **process algebraical semantics** (representation of the observed distributed system)

⇒ That contains **dynamic and epistemic modalities**

⇒ and operators for expressing the **modular features** of the system  
**Spatial Logics** – Cardelli, Gordon (2000), Cardelli, Caires (2003)

⇒ The agents observe subsystems and their dynamics; an agent can observe a process more or less depending on his observational abilities

⇒ For the agents, the observed processes are not only black boxes, but structurally organized

⇒ The society of agents has an algebraical structure that reflects the relation between the subsystems/processes they observe





$$K_A \langle \alpha \rangle \top$$

$$K_B \langle \alpha \rangle \top$$

$$K_E(K_A \langle \alpha \rangle \top \wedge K_B \langle \alpha \rangle \top) \rightarrow K_E \langle \alpha \rangle \top$$

$$K_E(K_A \langle \alpha \rangle \top \parallel K_B \langle \alpha \rangle \top) \rightarrow K_E(\langle \alpha \rangle \top \parallel \langle \alpha \rangle \top) \rightarrow K_E \langle \alpha \rangle \langle \alpha \rangle \top$$
$$E = A \parallel B$$

Society of agents over  $\mathbb{A}$  is a subset  $\mathfrak{A}$  of the set generated by

$$E := e \mid \alpha.E \mid E \parallel E \quad \alpha \in \mathbb{A}$$

s.t.  $e \in \mathfrak{A}$  (atomic agent)

if  $A \parallel B \in \mathfrak{A}$  then  $A, B \in \mathfrak{A}$

if  $\alpha.A \in \mathfrak{A}$  then  $A \in \mathfrak{A}$

The agents are unique up to commutativity of  $\parallel$ .

$$(\mathfrak{A}, e, \parallel, \alpha.)_{\alpha \in \mathbb{A}}$$





## Overview on Process Algebra



### CCS processes

Let  $\mathbb{A}$  be a denumerable set of *actions*.

an involution  $\bar{\cdot} : \mathbb{A} \longrightarrow \mathbb{A}$

associates to each  $\alpha \in \mathbb{A}$  the element  $\bar{\alpha} \in \mathbb{A}$  such that  $\overline{\bar{\alpha}} = \alpha$ .

The class  $\mathbb{P}$  of *processes* is given inductively by

$$P := 0 \mid \alpha.P \mid P \parallel P, \text{ where } \alpha \in \mathbb{A}.$$

Structural congruence on  $\mathbb{P}$

$\equiv \subseteq \mathbb{P} \times \mathbb{P}$  the smallest congruence relation on the structure of  $\mathbb{P}$  s.t.

$(\mathbb{P}, \parallel, 0)$  is a commutative monoid with respect to  $\equiv$ .

(congruence) if  $P' \equiv P''$  then 
$$\begin{cases} \alpha.P' \equiv \alpha.P'' \text{ for any } \alpha \in \mathbb{A} \\ P' \parallel P \equiv P'' \parallel P \text{ for any } P \in \mathbb{P}. \end{cases}$$



## Overview on Process Algebra



Labeled transition system on  $\mathbb{P}$

- $$\frac{}{\alpha.P \xrightarrow{\alpha} P}$$
- $$\frac{P \equiv Q \quad P \xrightarrow{\alpha} P'}{Q \xrightarrow{\alpha} P'}$$
- $$\frac{P \xrightarrow{\alpha} P' \quad Q \xrightarrow{\bar{\alpha}} Q'}{P \parallel Q \xrightarrow{(\alpha, \bar{\alpha})} P' \parallel Q'}$$
- $$\frac{P \xrightarrow{\alpha} P'}{P \parallel Q \xrightarrow{\alpha} P' \parallel Q}$$

We denote

$P \longrightarrow Q$  if  $P \xrightarrow{\alpha} Q$  or  $P \xrightarrow{(\alpha, \bar{\alpha})} Q$  for some  $\alpha \in \mathbb{A}$ .

$P \xrightarrow{Q:\alpha} P'$  whenever  $P \equiv Q \parallel R$ ,  $P' \equiv Q' \parallel R$  and  $Q \xrightarrow{\alpha} Q'$ .

$P \parallel Q \xrightarrow{P,Q:\alpha} P' \parallel Q'$  whenever  $P \xrightarrow{P:\alpha} P'$  and  $Q \xrightarrow{Q:\bar{\alpha}} Q'$ .

$$\mathbb{A}^+ = \{\alpha, P:\alpha, P,Q:\alpha \mid \alpha \in \mathbb{A} \text{ and } P, Q \in \mathbb{P}\} \ni l$$



## Kripke frames on CCS processes



Given a family of agents  $(\mathfrak{A}, e, \parallel, \alpha.)$  and a morfism

$$i : (\mathfrak{A}, e, \parallel, \alpha.) \rightarrow (\mathbb{P}, 0, \parallel, \alpha.),$$

a frame over  $i$  is

$$\mathcal{M}_i = \langle \Pi, \mathcal{R}_{\parallel}, \mathcal{R}_l, \mathcal{R}_A \rangle \text{ for } l \in \mathbb{A}^+ \text{ and } A \in \mathfrak{A}$$

$$\Pi = i(\mathfrak{A})$$

$$\mathcal{R}_{\parallel} \subseteq \Pi \times \Pi \times \Pi, (P, Q, R) \in \mathcal{R}_{\parallel} \text{ iff } P \equiv Q \parallel R$$

$$\mathcal{R}_l \subseteq \Pi \times \Pi, (P, Q) \in \mathcal{R}_l \text{ iff } P \xrightarrow{l} Q$$

$$\mathcal{R}_A \subseteq \Pi \times \Pi, (P, Q) \in \mathcal{R}_A \text{ iff } P \equiv i(A) \parallel P' \text{ and } Q \equiv i(A) \parallel Q'$$





## Syntax

*Let  $\mathfrak{A}$  be a society of epistemic agents*

$$\phi ::= 0 \mid \neg\phi \mid \phi \wedge \phi$$

## Process Semantics

$$\mathcal{M}_i = \langle \Pi, \mathcal{R}_{\parallel}, \mathcal{R}_l, \mathcal{R}_A \rangle \quad P \in \Pi$$

$$\mathcal{M}, P \models 0 \text{ iff } P \equiv 0$$

$$\mathcal{M}, P \models \neg\phi \text{ iff } \mathcal{M}, P \not\models \phi$$

$$\mathcal{M}, P \models \phi \wedge \psi \text{ iff } \mathcal{M}, P \models \phi \text{ and } \mathcal{M}, P \models \psi$$





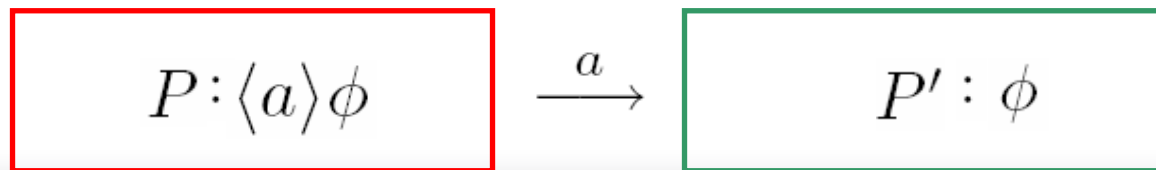
**Syntax**  $\phi := 0 \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle \phi$   
 $\langle a \rangle := \langle \alpha \rangle \mid \langle \alpha, \bar{\alpha} \rangle \mid \langle A : \alpha \rangle \mid \langle A, A' : \alpha \rangle.$

**Process Semantics**  $\mathcal{M}_i = \langle \Pi, \mathcal{R}_{\parallel}, \mathcal{R}_l, \mathcal{R}_A \rangle \quad P \in \Pi$

$$\mathcal{M}, P \models \langle a \rangle \phi \text{ iff } \begin{cases} P \xrightarrow{a} P' \\ \mathcal{M}, P' \models \phi \end{cases}$$

$$\mathcal{M}, P \models \langle A : \alpha \rangle \phi \text{ iff } \begin{cases} P \xrightarrow{i(A):\alpha} P' \\ \mathcal{M}, P' \models \phi \end{cases}$$

$$\mathcal{M}, P \models \langle A, A' : \alpha \rangle \phi \text{ iff } \begin{cases} P \xrightarrow{i(A), i(A'):\alpha} P' \\ \mathcal{M}, P' \models \phi \end{cases}$$

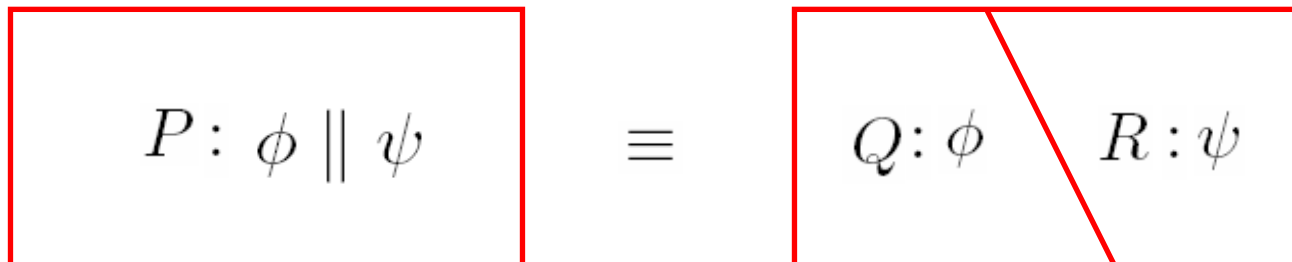




**Syntax**  $\phi := 0 \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle\phi \mid \phi \parallel \phi$

**Process Semantics**  $\mathcal{M}_i = \langle \Pi, \mathcal{R}_{\parallel}, \mathcal{R}_l, \mathcal{R}_A \rangle \quad P \in \Pi$

$\mathcal{M}, P \models \phi \parallel \psi$  iff  $P \equiv Q \parallel R$  and  $\begin{cases} \mathcal{M}, Q \models \phi \\ \mathcal{M}, R \models \psi \end{cases}$   
 $(P, Q, R) \in \mathcal{R}_{\parallel}$





## Syntax

$$\phi := 0 \mid \neg\phi \mid \phi \wedge \phi \mid \langle a \rangle\phi \mid \phi \parallel \phi \mid K_A\phi$$

$K_A\phi$  “agent  $A$  knows  $\phi$ ”

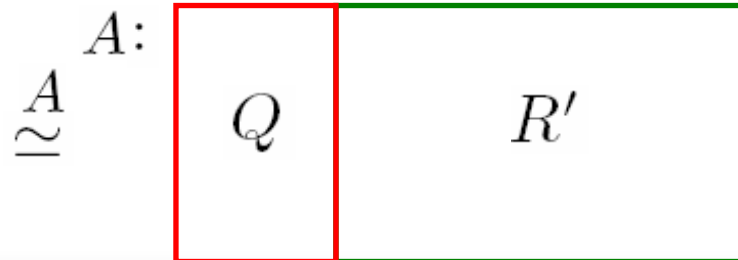
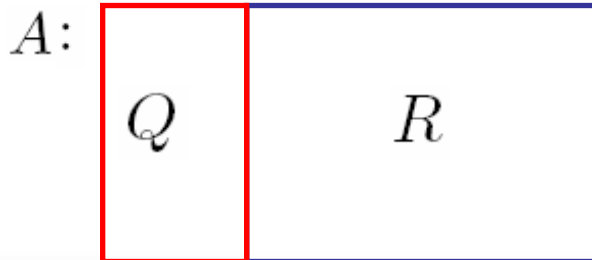
$K_{A_1}K_{A_2}\phi$  “agent  $A_1$  knows that agent  $A_2$  knows  $\phi$ ”

**Process Semantics**  $\mathcal{M}_i = \langle \Pi, \mathcal{R}_{\parallel}, \mathcal{R}_l, \mathcal{R}_A \rangle \quad P \in \Pi$

$$\mathcal{M}, P \models K_A\phi \text{ iff } \begin{cases} \forall Q \in \Pi \text{ s.t. } (P, Q) \in \mathcal{R}_A, \mathcal{M}, Q \models \phi \\ P \equiv i(A) \parallel P' \\ \forall i(A) \parallel Q' \in \Pi, \mathcal{M}, i(A) \parallel Q' \models \phi \end{cases}$$

$$P \equiv Q|R : \phi$$

$$Q|R' : \phi$$





## Derived operators

$$1 \stackrel{def}{=} \neg((\neg 0) \parallel (\neg 0))$$

$$\alpha.\psi \stackrel{def}{=} (\langle \alpha \rangle \psi) \wedge 1$$

$$\tilde{K}_A \phi \stackrel{def}{=} \neg K_A \neg \phi$$

$$\mathcal{M}, P \models 1 \text{ iff } P \equiv 0 \text{ or } P \equiv \alpha.Q$$

$$\mathcal{M}, P \models \alpha.\phi \text{ iff } P \equiv \alpha.Q \text{ and } \mathcal{M}, Q \models \phi$$

$$\mathcal{M}, P \models \tilde{K}_A \phi \text{ iff } \begin{cases} \text{either } P \not\equiv I(A) \parallel R \\ \text{or } \exists I(A) \parallel S \in \Pi, \mathcal{M}, I(A) \parallel S \not\models \phi \end{cases}$$





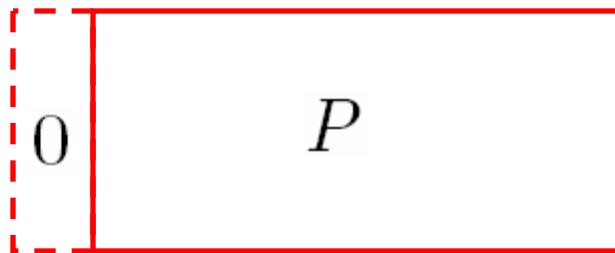
## Universal modality



Consider the atomic agent  $e \in \mathcal{A}$

$$\mathcal{M}, P \models K_e \phi \text{ iff } \begin{cases} P \equiv P|0 \\ \forall Q \in \Pi, \quad \mathcal{M}, Q \models \phi. \end{cases}$$

$$\mathcal{M}, P \models \tilde{K}_e \phi \text{ iff } \begin{cases} \text{either } P \not\equiv P|0 & \text{not possible} \\ \text{or } \exists Q \in \Pi \quad \mathcal{M}, Q \models \phi. \end{cases}$$



$$P \equiv P|0$$

$e$



$$Q \equiv Q|0$$



## Characteristic Formulas



$$\mathcal{F}_{\mathbb{P}} = \{(f_P) \mid P \in \mathbb{P}\}$$

$$f_0 \stackrel{def}{=} 0 \quad f_{P|Q} \stackrel{def}{=} f_P|f_Q \quad f_{\alpha.P} \stackrel{def}{=} \alpha.f_P$$

**Proposition**  $\mathcal{M}, P \models f_Q$  iff  $P \equiv Q$ .

Let  $\Phi$  be a finite set of formulas

$$\Delta\Phi \stackrel{def}{=} K_e(\bigvee_{\phi \in \Phi} \phi) \wedge (\bigwedge_{\phi \in \Phi} \tilde{K}_e\phi).$$

$\mathcal{M}, P \models \Delta\Phi$  iff

for any  $Q \in \Pi$  there exists  $\phi \in \Phi$  such that  $\mathcal{M}, Q \models \phi$

for any  $\phi \in \Phi$  there exists  $Q \in \Pi$  such that  $\mathcal{M}, Q \models \phi$ .





## Spatial axioms

$$S1: \vdash \top | \perp \rightarrow \perp$$

$$S2: \vdash (\phi | \psi) | \rho \rightarrow \phi | (\psi | \rho)$$

$$S3: \vdash \phi | 0 \leftrightarrow \phi$$

$$S4: \vdash \phi | (\psi \vee \rho) \rightarrow (\phi | \psi) \vee (\phi | \rho)$$

$$S5: \vdash \phi | \psi \rightarrow \psi | \phi$$

## Spatial rules

$$SR1: \vdash \phi \rightarrow \psi \text{ then } \vdash \phi | \rho \rightarrow \psi | \rho$$





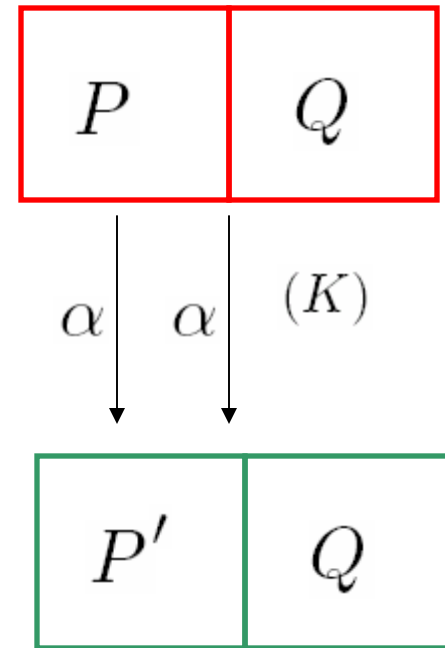
## Dynamic axioms

$$D1: \vdash \langle a \rangle \phi | \psi \rightarrow \langle a \rangle (\phi | \psi)$$

$$D2: \vdash [a] (\phi \rightarrow \psi) \rightarrow ([a] \phi \rightarrow [a] \psi)$$

$$D3: \vdash 0 \vee \alpha. \top \rightarrow [\beta] \perp$$

$$D4: \vdash \alpha. \phi \rightarrow [\alpha] \phi$$



## Dynamic rules

$$DR1: \text{If } \vdash \phi \text{ then } \vdash [a] \phi$$

*(Nec)*

$$DR2: \text{If } \vdash \phi_1 \rightarrow [a] \phi'_1 \text{ and } \vdash \phi_2 \rightarrow [a] \phi'_2 \\ \text{then } \vdash \phi_1 | \phi_2 \rightarrow [a] (\phi'_1 | \phi_2 \vee \phi_1 | \phi'_2)$$





## Epistemic axioms

$$\text{E12: } \vdash K_A\phi \wedge K_A(\phi \rightarrow \psi) \rightarrow K_A\psi \quad (K)$$

$$\text{E13: } \vdash K_A\phi \rightarrow \phi \quad (T)$$

$$\text{E14: } \vdash K_A\phi \rightarrow K_AK_A\phi$$

$$\text{E15: } \vdash K_A\top \rightarrow (\neg K_A\phi \rightarrow K_A\neg K_A\phi) \quad (5)$$

## Epistemic rules

$$\text{ER5: } \vdash \phi \text{ then } \vdash K_A\top \rightarrow K_A\phi \quad (Nec)$$





## Mixed axioms

$$\text{M3: } \vdash \langle A : \alpha \rangle \phi \wedge \langle A|A' : \alpha \rangle \top \rightarrow \langle A|A' : \alpha \rangle \phi$$

$$\text{M4: } \vdash \langle A : \alpha \rangle \phi | \langle A' : \bar{\alpha} \rangle \psi \rightarrow \langle A, A' : \alpha \rangle (\phi | \psi)$$

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**Theorem**  $\vdash K_A \phi \rightarrow K_A \top$

**Theorem (Monotonicity of knowledge)**

$$\text{If } \vdash \phi \rightarrow \psi \text{ then } \vdash K_A \phi \rightarrow K_A \psi$$

**Theorem (Consistency of knowledge)**

$$\vdash K_A \phi \rightarrow \neg K_A \neg \phi$$



## Metatheorems



### Metatheorem (Soundness)

*The system  $\mathcal{L}_{\mathbb{A}}^{\mathcal{A}}$  is sound w.r.t. process semantics.*

### Metatheorem (Completeness)

*The system  $\mathcal{L}_{\mathbb{A}}^{\mathcal{A}}$  is complete w.r.t. process semantics.*

### Metatheorem (Finite model property)

*The system  $\mathcal{L}_{\mathbb{A}}^{\mathcal{A}}$  has the finite model property w.r.t. process semantics.*

### Metatheorem (Decidability)

*For  $\mathcal{L}_{DES}$  validity, satisfiability and model checking are decidable against the process semantics.*



## Potential applications



- Secure Communication => checking secrecy and authentication for given communication protocols in contextual situations
- Artificial Intelligence => endowing artificial agents with good and flexible tools to reason about their changing environment and about each other
- Semantic Web => in designing strategies for knowledge acquisition over complex networks (such as the Internet)
- Debugging and Diagnosis => identifying the agents that generates errors
- Performance Analysis => identifying the “expensive” agent
- Systems Biology => biological agents are membrane-bounded and mobile chemical soups; they interact by exchanging chemical elements







- A new logic for knowledge in Multiagent-Systems
  - the semantic is based on process algebra
  - agents have (ontological) structures
  - agents can “develop knowledge” about dynamic distributed systems
  - agents are individuals, societies of individuals, societies of societies
  - agents are active: they can update information (knowledge) and can change their spatial position
  - has a sound and complete Hilbert-style axiomatic system
  - validity, satisfiability and model-checking are decidable

