INF223 Categories for Software Engineering
Spring 2006

— Exercise Set 4 , for group meeting 17.02.2006 —

1. (a) By **2** we want to denote the following graph

$$\begin{tikzcd} 1 \arrow[r, bend left, "a"] & 2 \arrow[l, bend left, "b"] \end{tikzcd}$$

   Give an interpretation of the objects in the slice category $\mathbf{GRAPH}{\downarrow}\mathbf{2}$.

   (b) Define a graph G with $G_0 = \{red, blue, green\}$ such that in the slice category $\mathbf{GRAPH}{\downarrow}\, G$ of G-typed graphs we have only arrows from "red nodes" to "red nodes", from "red nodes" to "blue nodes" and from "green nodes" to "blue nodes".

2. Let $\mathbf{D}$ be a subcategory of $\mathbf{C}$. Validate the following statements:

   (a) Two objects that are isomorphic in $\mathbf{D}$ are necessarily isomorphic in $\mathbf{C}$.

   (b) If $\mathbf{D}$ is a full subcategory of $\mathbf{C}$ then two objects that are isomorphic in $\mathbf{C}$ are also isomorphic in $\mathbf{D}$.

3. For a function $f : A \to B$ we obtain a function $f^{-1} : \wp(B) \to \wp(A)$ in the inverse direction between the corresponding power sets by

$$f^{-1}(B_0) =_{def} \{a \in A \mid f(a) \in B_0\}$$

   Show that this *formation of inverse images* is compatible with composition and identities. That is, show that

$$(f;g)^{-1} = g^{-1}; f^{-1}$$

   for all functions $f : A \to B$, $g : B \to C$ and

$$id_A^{-1} = id_{\wp(A)} \quad \text{for all sets } A.$$