A COMBINATORIAL PROOF OF THE DIVISIBILITY OF CERTAIN ENTRIES IN NUMERICAL TRIANGLES

by Solomon W. Golomb
University of Southern California

PLENARY LECTURE

NINTH NORDIC COMBINATORIC CONFERENCE

BERGEN, NORGE (NOREG)

(SELMER SENTERET VED

UNIVERSITETET I BERGEN)

NOVEMBER 21-22, 2007

A COMBINATORIAL PROOF OF THE DIVISIBILITY OF CERTAIN ENTRIES IN NUMERICAL TRIANGLES

– Solomon W. Golomb

Abstract

The most famous result of the type to be discussed in this paper is that the binomial coefficient $\binom{p}{k}$ is divisible by p for all $k, 0 < k < p$, whenever p is prime. This is almost always proved as a consequence of Gauss's Lemma in Number Theory that if a prime p divides a product ab, then p must divide either a or b (or both). However, it is also true that the Stirling number of the first kind, $s(p,k)$, and the Stirling number of the second kind, $S(p,k)$, is divisible by p for all $k, 1 < k < p$, whenever p is prime, but there is no corresponding proof from Number Theory. Instead, we give a simple combinatorial approach that can be used to show that the "interior" entries in the p^{th} row of these three numerical triangles, and several others as well, must be divisible by p whenever p is prime. The combinatorial proof shows that the objects being counted occur in disjoint subsets of size p, so that their total number must be a multiple of p.