Trær

I. Eksempler, Definisjon

II. Binære Træ

III. ADT Tre

IV. Basis TreAlgoritmer (traversering)

V. Implementasjon av Binære Træ

Lenket Struktur

Sequence (Array)

VI. Noen Anvendelser

Syntaks trær

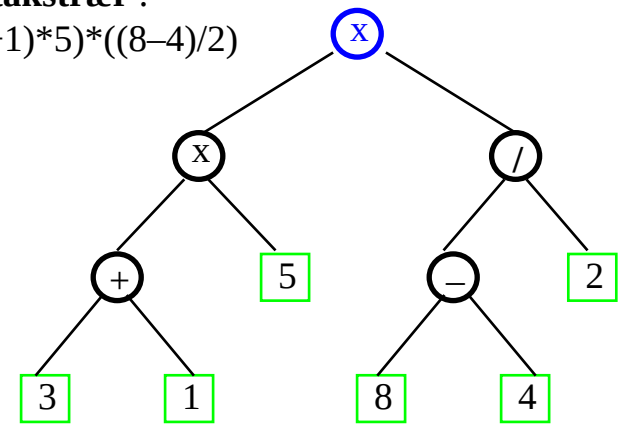
Ordbok Søking

Kap. 6 (kursorisk: 6.4.4; + DFS/BFS)

Noen eksempler

Syntakstrær :

((3+1)*5)*((8–4)/2)

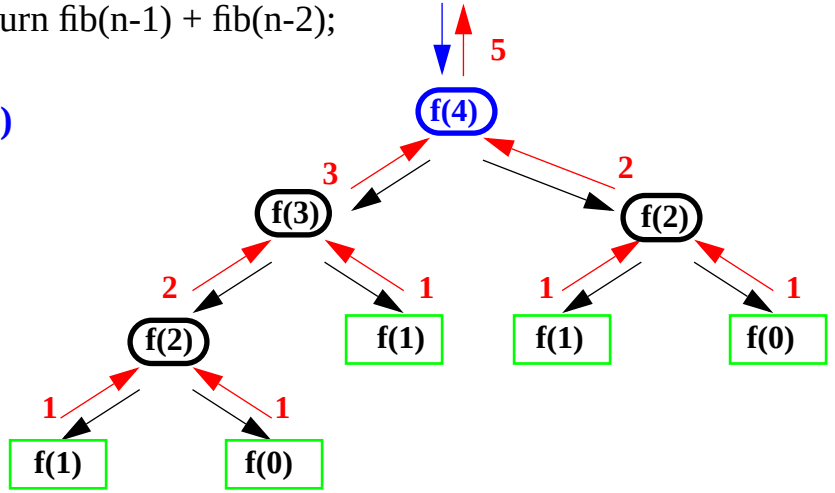


((3 + 1) x 5) x ((8 – 4) / 2)

Trær av rekursive kall :

```
int fib(int n) {
  if (n==0 || n==1) return 1;
  else
      return fib(n-1) + fib(n-2);
}
```

> **fib(4)**



Filhierarki :

> **cd**

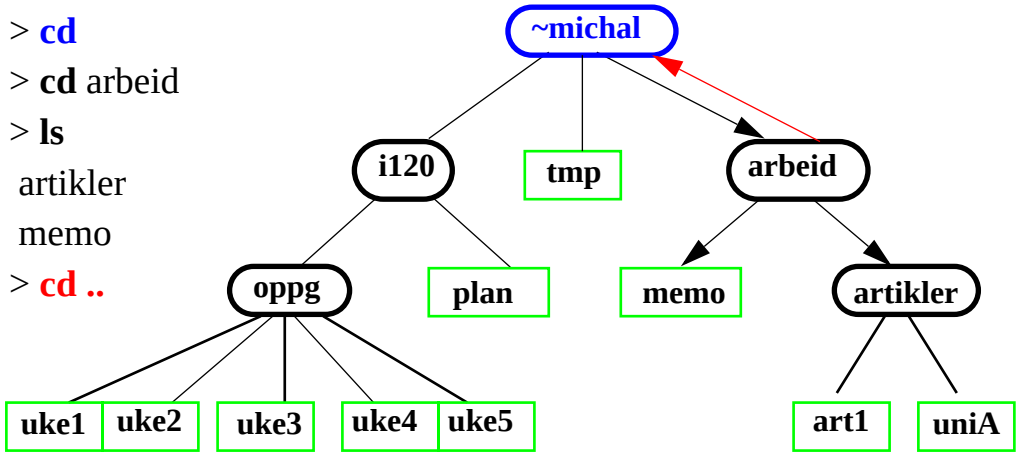
> **cd** arbeid

> **ls**

artikler

memo

> **cd ..**



et **Tre** T er enten :

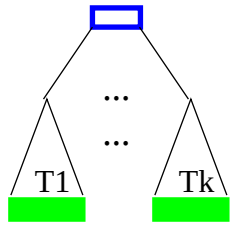
en enkelt node

eller består av

en *node r*

(r rot-node) og

k subtrær



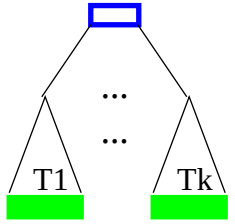
Terminologi (fra slektskapstrær)

et **Tre** T er enten :

en enkelt node r

eller

en rot-node r
og *k subtrær*



T i figuren har 12 noder (generellt ADT position)**, r** er **roten** i **T**

1. **b** er **foreldrenode** (parent) til **e**, **f**, **g** , og **forgjenger** til **e**, **f**, **g**, **j**, **k**
2. **e**, **f**, **g** er **barna** til **b** : umiddelbare etterfølgere (og disse er **søsken**)
3. **d**, **f**, **g**, **h**, **i**, **j**, **k** **eksterne** noder (**løv**) : har ingen barn
4. **r**, **a**, **b**, **c**, **e** er **interne** noder : ikke løv
5. **dybden** av **e** = 2 : lengden av stien fra roten (**nivå**)

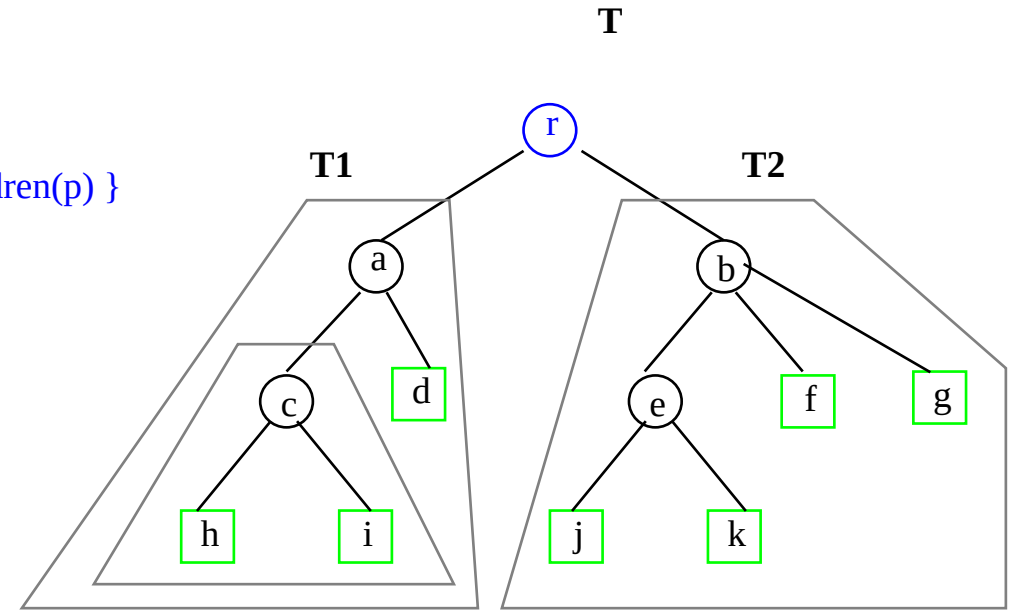
   depth(p) = if isRoot(p) return 0
   else return 1+ depth(parent(p))
6. **høyden** av **b** = 2 : avstand til fjerneste løv under b

   height(p) = if isExternal(p) return 0
   else return 1+ max{ height(x) : x er et av children(p) }

   høyden av **T** = høyden av roten til T
7. **grad** av **b** = 3 : antall barn
8. **T1**, **T2** er **deltrær** av **T**

Noen egenskaper

9. antall kanter = antall noder – 1
10. hver node har en *entydig sti* til roten



2. Binære Trær

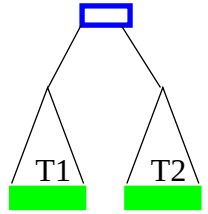
-hver node har **2** eller **0 barn**
-barna er ordnet: **venstre og høyre**

et **Binært Tre** er enten :

en *node* r

eller

en *node* r
og
2 *binære subtrær*
(venstre og høyre)



av og til : **høyst 2** binære subtrær

Noen egenskaper (h: høyde; n: antall noder = # noder)

A. # eksterne noder = # interne noder +1

B. # noder på nivå i $\leq 2^i$

C. $h+1 \leq$ (# eksterne noder) $\leq 2^h$

$\log_2$ (# eksterne noder) $\leq h$

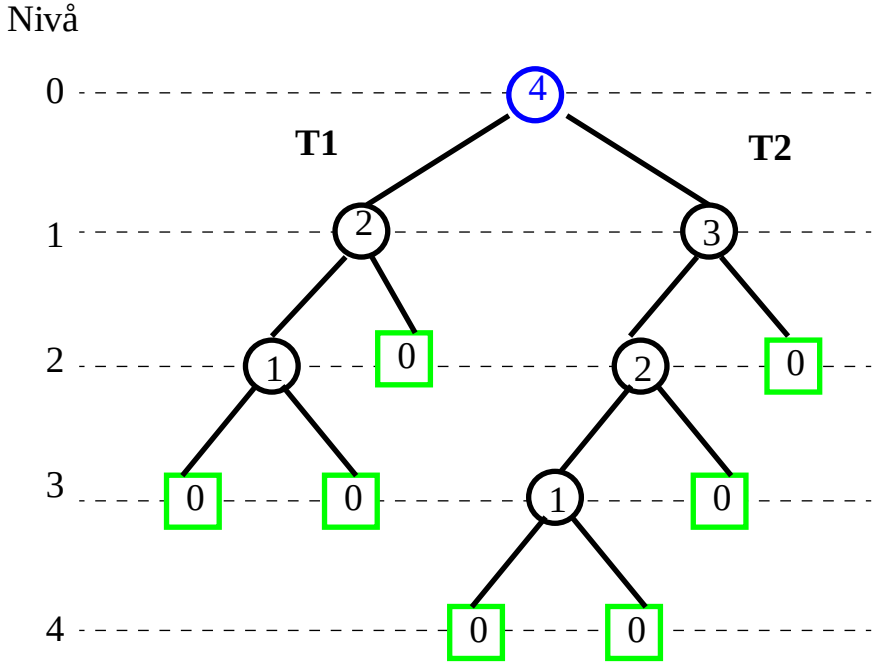
D. $h \leq$ (# interne noder) $\leq 2^h - 1$

$\log_2$ (# interne noder) $\leq h$

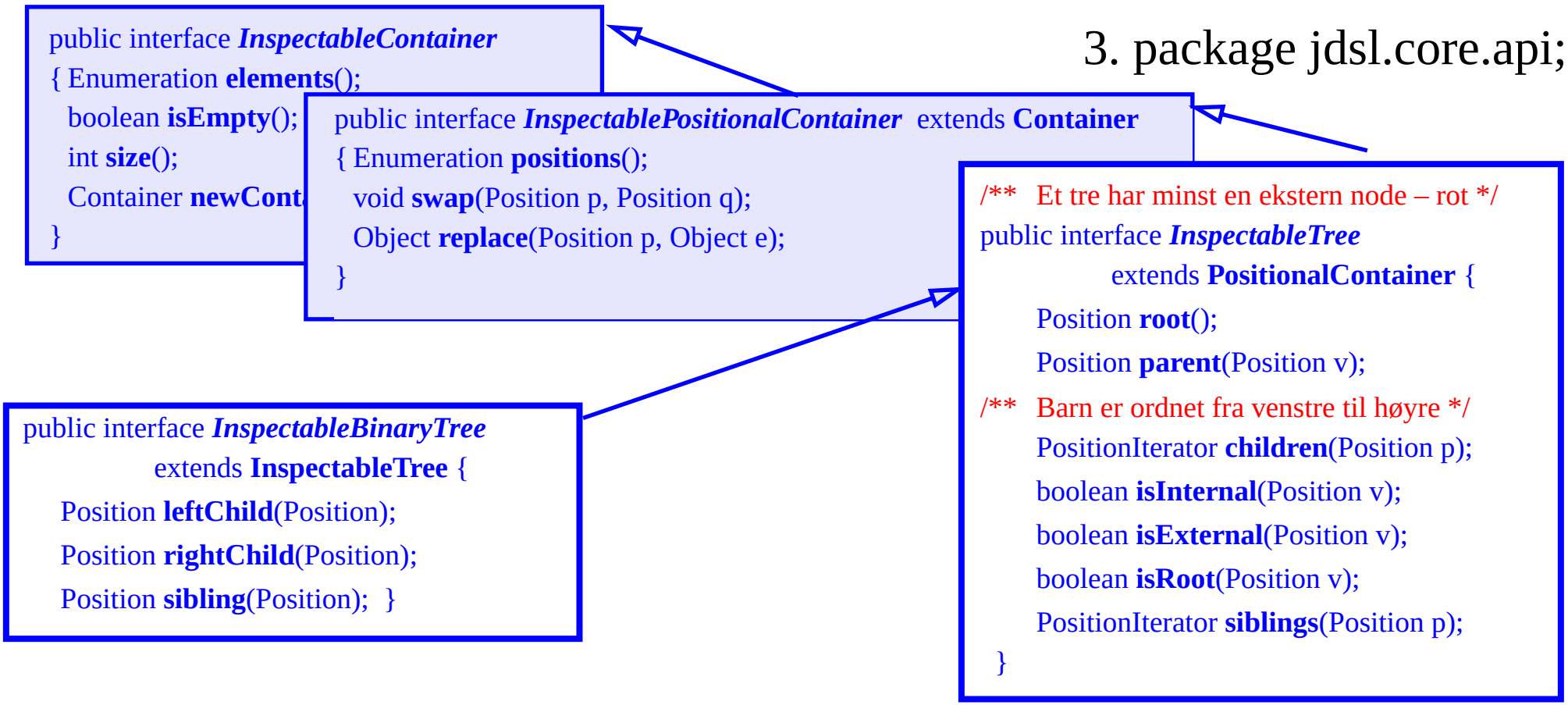
E. $2h+1 \leq n \leq 2^{(h+1)} - 1$

$\log_2(n+1) - 1 \leq h \leq (n-1)/2$

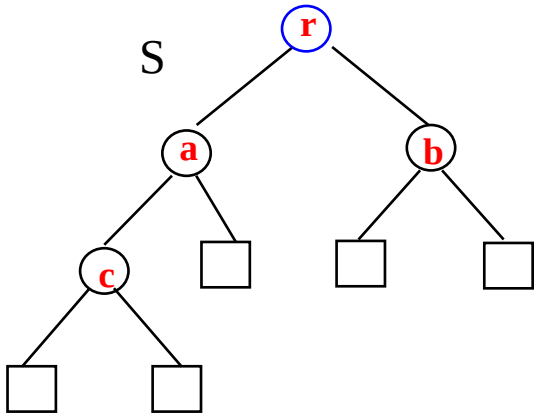
T

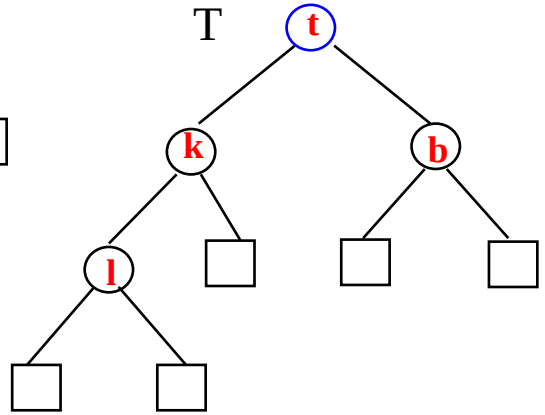


3. package jdsl.core.api;



BinaryTree: likhet vs. isomorfisme

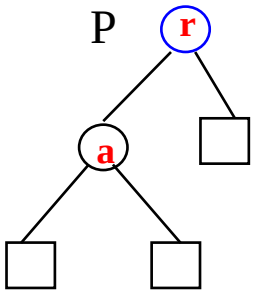


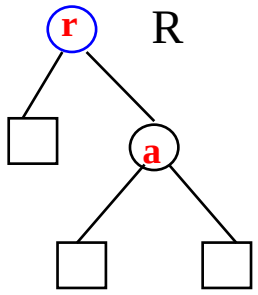



```
boolean iso (InspectableBinaryTree P)
{   return iso(root, P.root(), P) ;  }

boolean equals (InspectableBinaryTree P)
{   return equ(root, P.root(), P) ;  }

boolean iso // equ
        (Position r, Position p, InspectableBinaryTree P)
{   if (isExternal(r) && P.isExternal(p))
      return true; // r.element().equals(p.element());
    else if (isInternal(r) && P.isInternal(p) )
      return ( // r.element().equals(p.element())  &&
          iso(leftChild(r), P.leftChild(p), P) &&          // equ
          iso(rightChild(r), P.rightChild(p), P) );        // equ
    else return false;
}
```

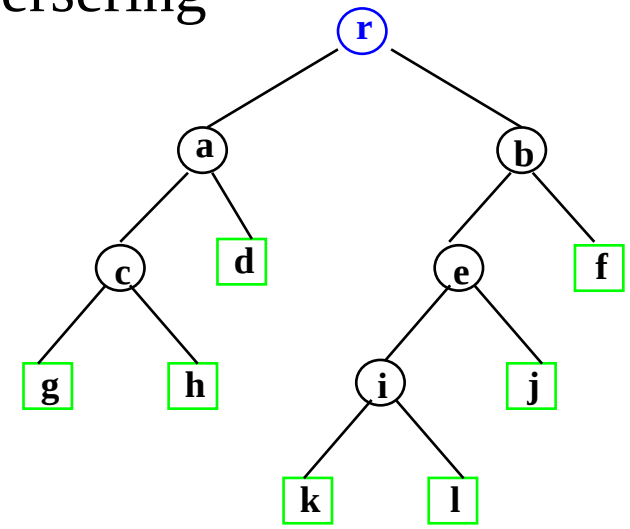


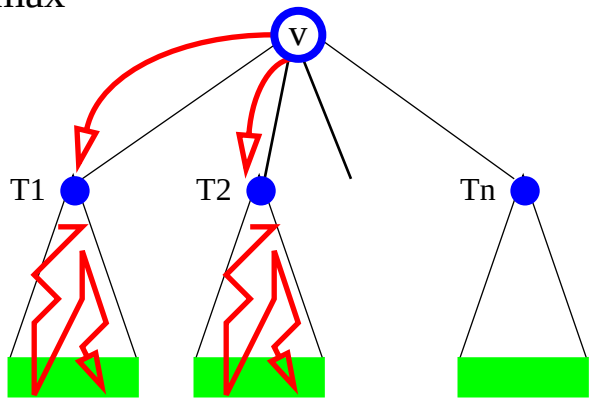


4. Tre-Algoritmer : traversering

PositionIterator positions() – *ok ... men i hvilken rekkefølge ?*

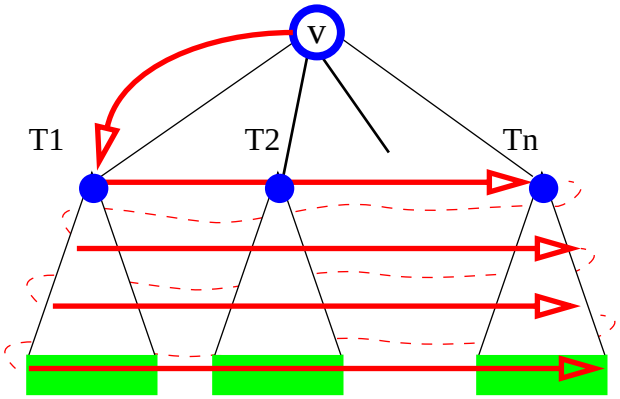
```
int height(Position v)
   if (isExternal(v)) return 0
   else // 1+max{ height(p): p i children(v) }
      max=0
      for hver p i children(v)
         h=height(p); if (h>max) max=h
      return 1+max
```






```
void DFS(Tree T, Position v) {
  for hver p i T.children(v)
      DFS(T,p)    }
```

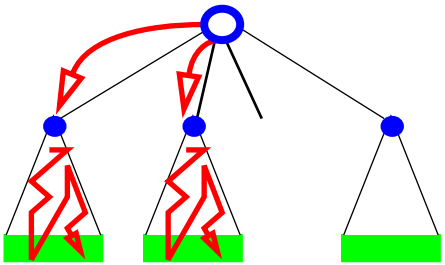
enumererer: **r, a,c,g,h,d, b, e,i,k,l,j, f**

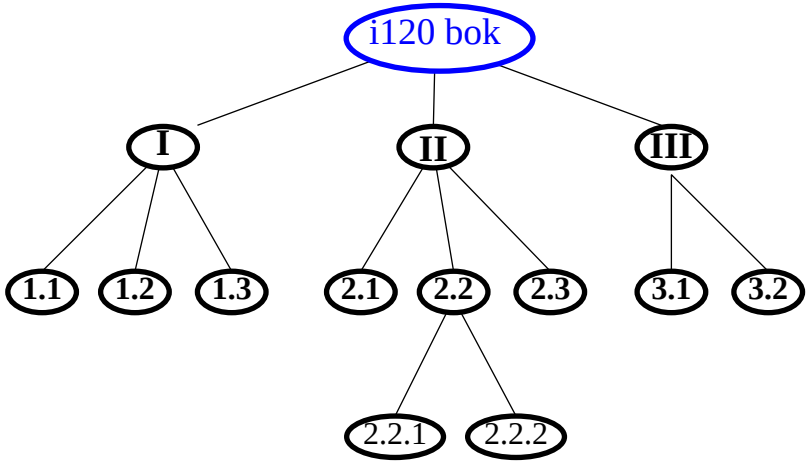


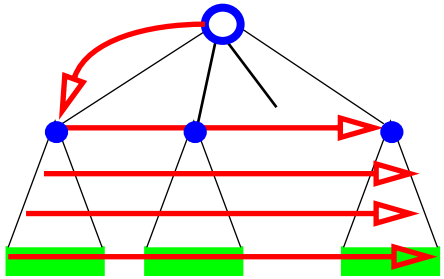
void **BFS**(Tree T, Position v)

enumererer: **r, a,b, c,d,e,f, g,h,i,j, k,l**

DFS vs. BFS







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Chap. I Design Principles

1.1 DS & Alg
1.2 OO-Programming
1.3 JAVA

Chap. II Analysis Tools

2.1 Alg. Analysis
2.2 Running Time
2.2.1 O-notation
2.2.2 Avarage Case
2.3 Worst Case

Chap. III Basic DS

3.1 Stacks
3.2 Queues

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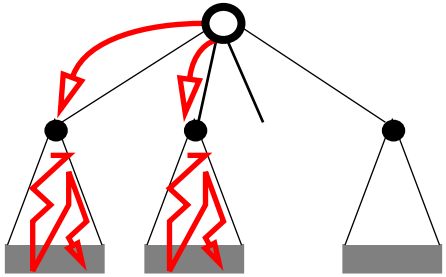
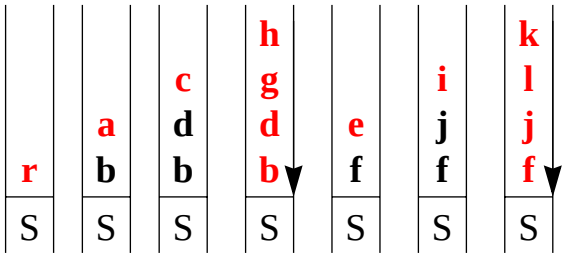
DFS

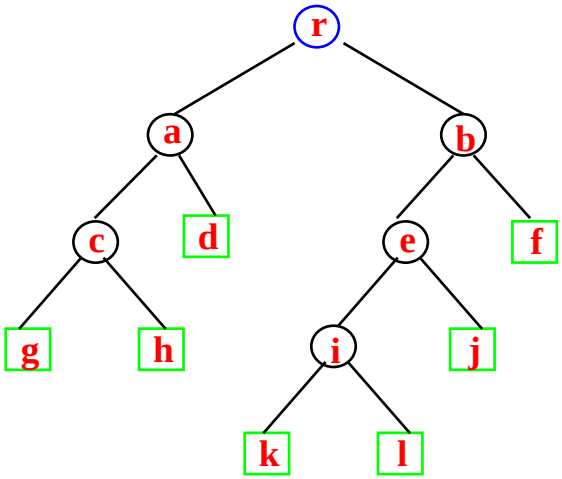


r, a,c, h,g, d, b, e,i,k,l, j, f

```
/*DFS(Tree T)
  *   Stack S = new StackIm()
  *   S.push(T.root())
  *   while (!S.isEmpty())
  *     p= S.pop()
  *     S.push(T.children(p))
  */
```



og



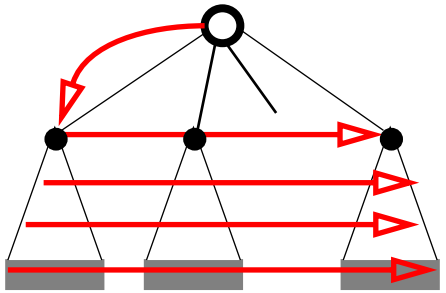
***"samme"** algoritme:*

```
trav(Tree T, LiFi S)
    S.add(T.root());
    while (! S.isEmpty())
        p = S.remove();
        S.add(T.children(p))
```

trav(T, new **StackIm**())

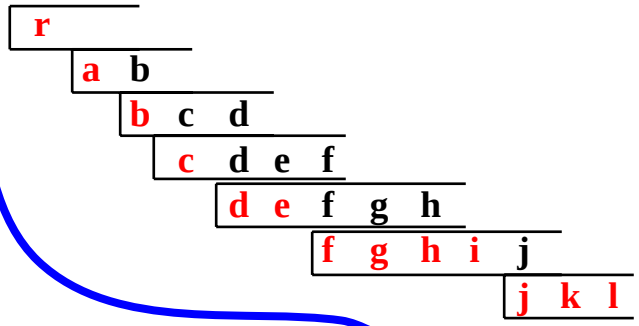
trav(T, new **QueueIm**())

BFS



r, a,b, c,d,e,f, g,h,i,j, k,l

```
/*BFS(Tree T)
  *   Queue S = new QueueIm()
  *   S.enqueue(T.root())
  *   while (!S.isEmpty())
  *       p= S.dequeue()
  *       S.enqueue(T.children(p))
  */
```



PreOrder

gjør jobben
FØR
rekursive kall

```
DFS(Tree T, Position v)
  Print(v.element())
  for hver p i T.children(v)
      DFS(T,p)
```

I-120 bok

Chap. I Design Principles
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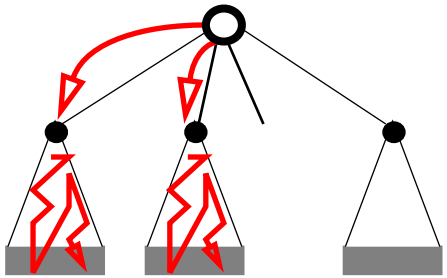
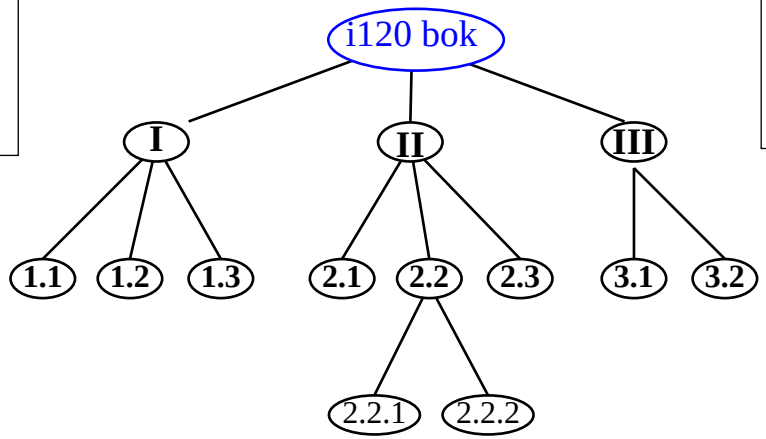
DFS





PostOrder

gjør jobben
ETTER
rekursive kall

```
DFS(Tree T, Position v)

  for hver p i T.children(v)
      DFS(T,p)
  Print(v.element())
```

- **1.1** DS & Alg
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Chap. I Design Principles
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Chap. II Analysis Tools
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Chap. III Basic DS

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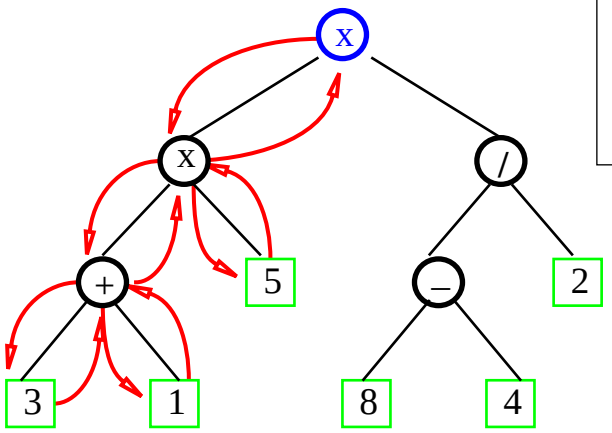
Binære Trær : **Euler-Tour & InOrder**

```
DFS(Tree T, Position v)
  ??? - pre
  for hver p i T.children(v)
      DFS(T,p)
  ??? - post
```

```
DFS(BinTree B, Position v)
  ??? – pre (left)
  if (isInternal(v))
      DFS(B,B.leftChild(v))
  ??? – in-order (below)
  if (isInternal(v))
      DFS(B,B.rightChild(v))
  ??? – post (right)
```

```
DFS(BinTree B, Position v)
  if (B.isExternal(v)) ??? – ekst
  else
      ??? – pre
      DFS(B,B.leftChild(v))
      ??? – in-order
      DFS(B,B.rightChild(v))
      ??? – post
```



((3 + 1) x 5) x ((8 – 4) / 2) = 40

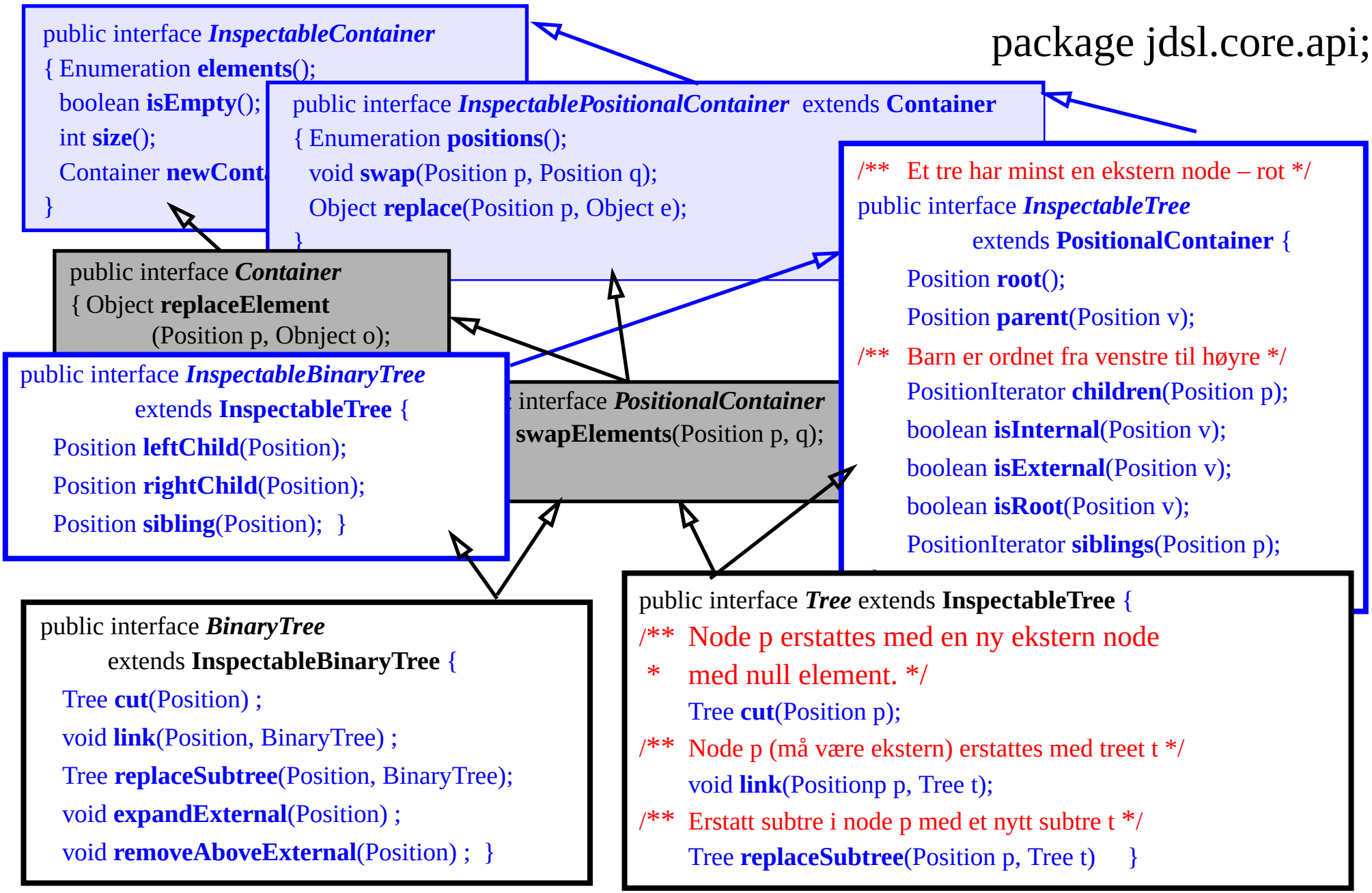
```
void Pr(BinTree B, Position v)
  if (B.isExternal(v)) print(v.element())
  else
      print("(")
      Pr(B,B.leftChild(v))
      print(v.element())
      Pr(B,B.rightChild(v))
      print(")")
```

```
print(v.element())
Pr(B,B.leftChild(v))
Pr(B,B.rightChild(v))
```

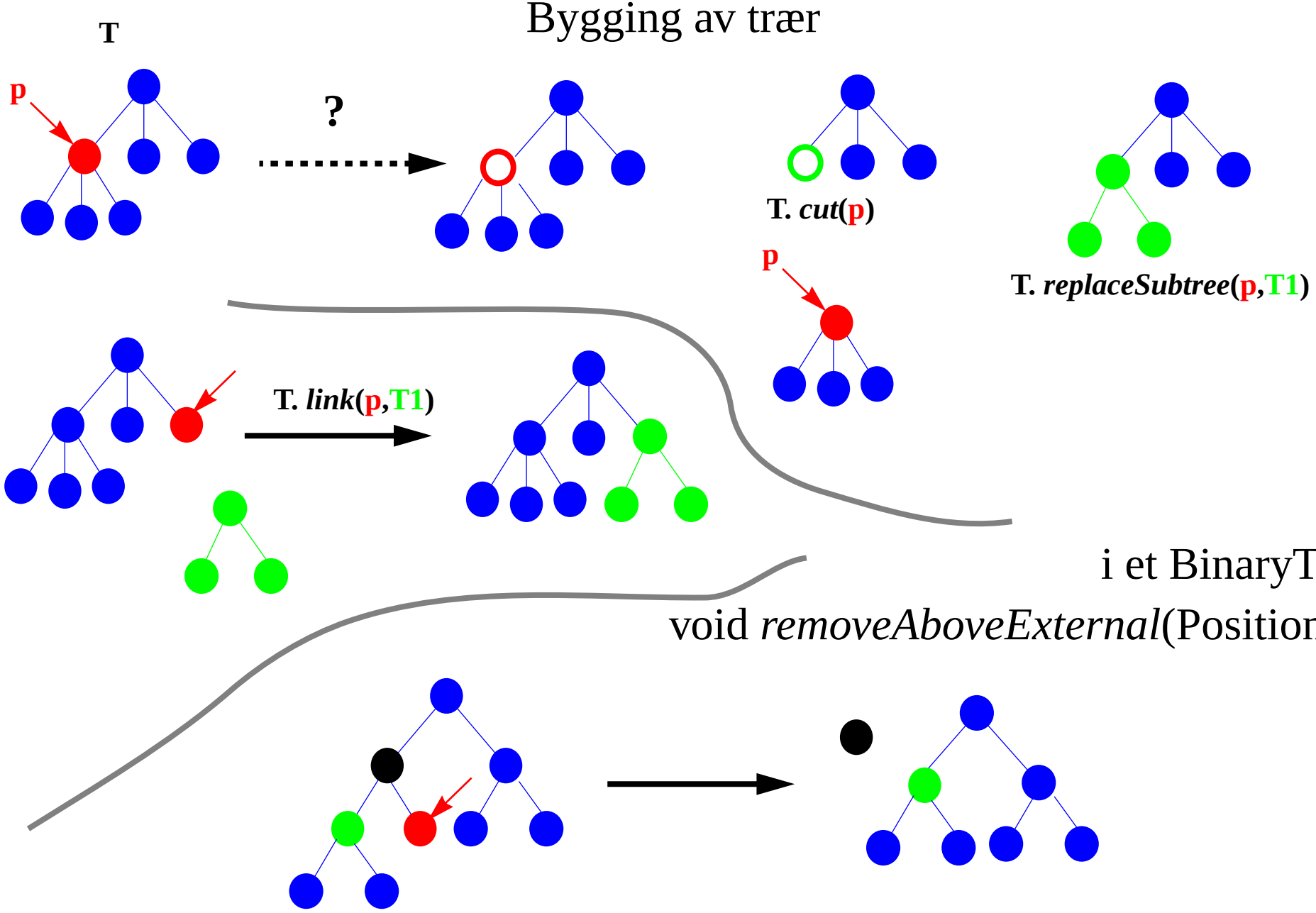
```
int val(BinTree B, Position v)
  if (B.isExternal(v))
    return integer(v)
  else
    L= val(B,B.leftChild(v))
    R= val(B,B.rightChild(v))
    return operasjon(v)(L,R))
```

x x + 3 1 5 / – 8 4 2

x [x(+ (3,1), 5), / (–(8,4),2)]


package jdsl.core.api;
public interface InspectableContainer
{ Enumeration elements();
boolean isEmpty();
int size();
Container newCont
}
public interface InspectablePositionalContainer extends Container
{ Enumeration positions();
void swap(Position p, Position q);
Object replace(Position p, Object e);
}
public interface Container
{ Object replaceElement
(Position p, Obnject o);
interface PositionalContainer
swapElements(Position p, q);
/** Et tre har minst en ekstern node – rot */
public interface InspectableTree
extends PositionalContainer {
Position root();
Position parent(Position v);
/** Barn er ordnet fra venstre til høyre */
PositionIterator children(Position p);
boolean isInternal(Position v);
boolean isExternal(Position v);
boolean isRoot(Position v);
PositionIterator siblings(Position p);
public interface InspectableBinaryTree
extends InspectableTree {
Position leftChild(Position);
Position rightChild(Position);
Position sibling(Position); }
public interface BinaryTree
extends InspectableBinaryTree {
Tree cut(Position) ;
void link(Position, BinaryTree) ;
Tree replaceSubtree(Position, BinaryTree);
void expandExternal(Position) ;
void removeAboveExternal(Position) ; }
public interface Tree extends InspectableTree {
/** Node p erstattes med en ny ekstern node
* med null element. */
Tree cut(Position p);
/** Node p (må være ekstern) erstattes med treet t */
void link(Positionp p, Tree t);
/** Erstatt subtre i node p med et nytt subtre t */
Tree replaceSubtree(Position p, Tree t) }

Bygging av trær

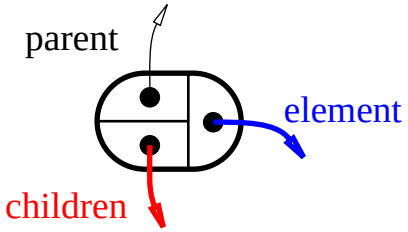


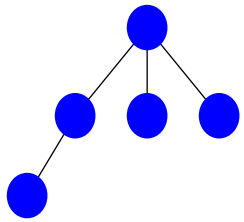
i et BinaryTree

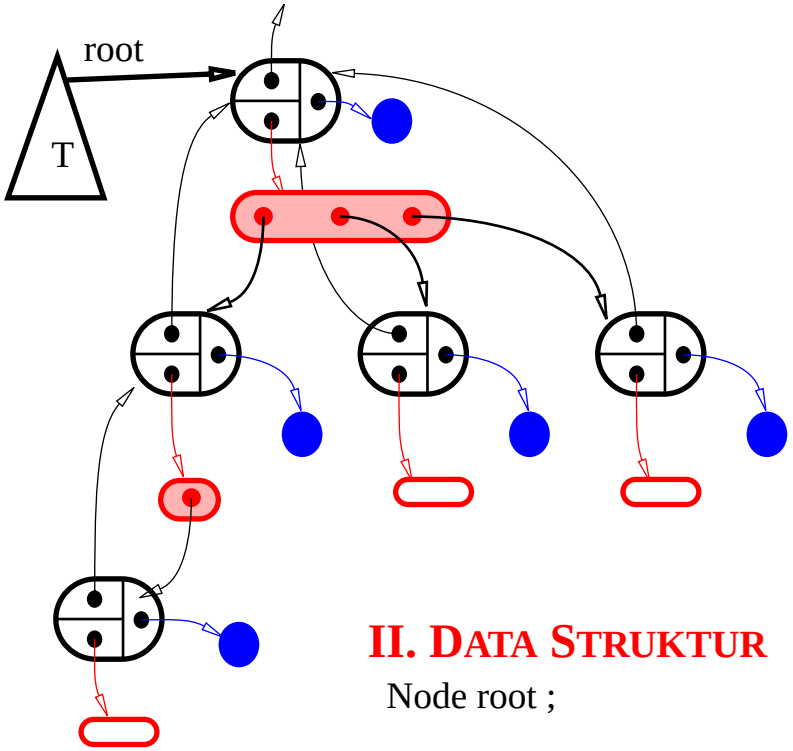
void *removeAboveExternal*(Position p)

5. Implementasjon av Tree – *med LenketStruktur*

I. DATA REPRESENTASJON







II. DATA STRUKTUR

Node root ;

III. DATA INVARIANT : *for alle Position* p, v *i* T:

- p.container() == T
- T.root() != null
- isRoot(p) ↔ (p.parent()==null)
- T.isEmpty() ↔ T.root().elem() == null
- isExternal(p) == p.children().isEmpty()
- isInternal(p) == !p.children().isEmpty()
- v.parent()==p ↔ v er bland p.children()
- p.children() != null

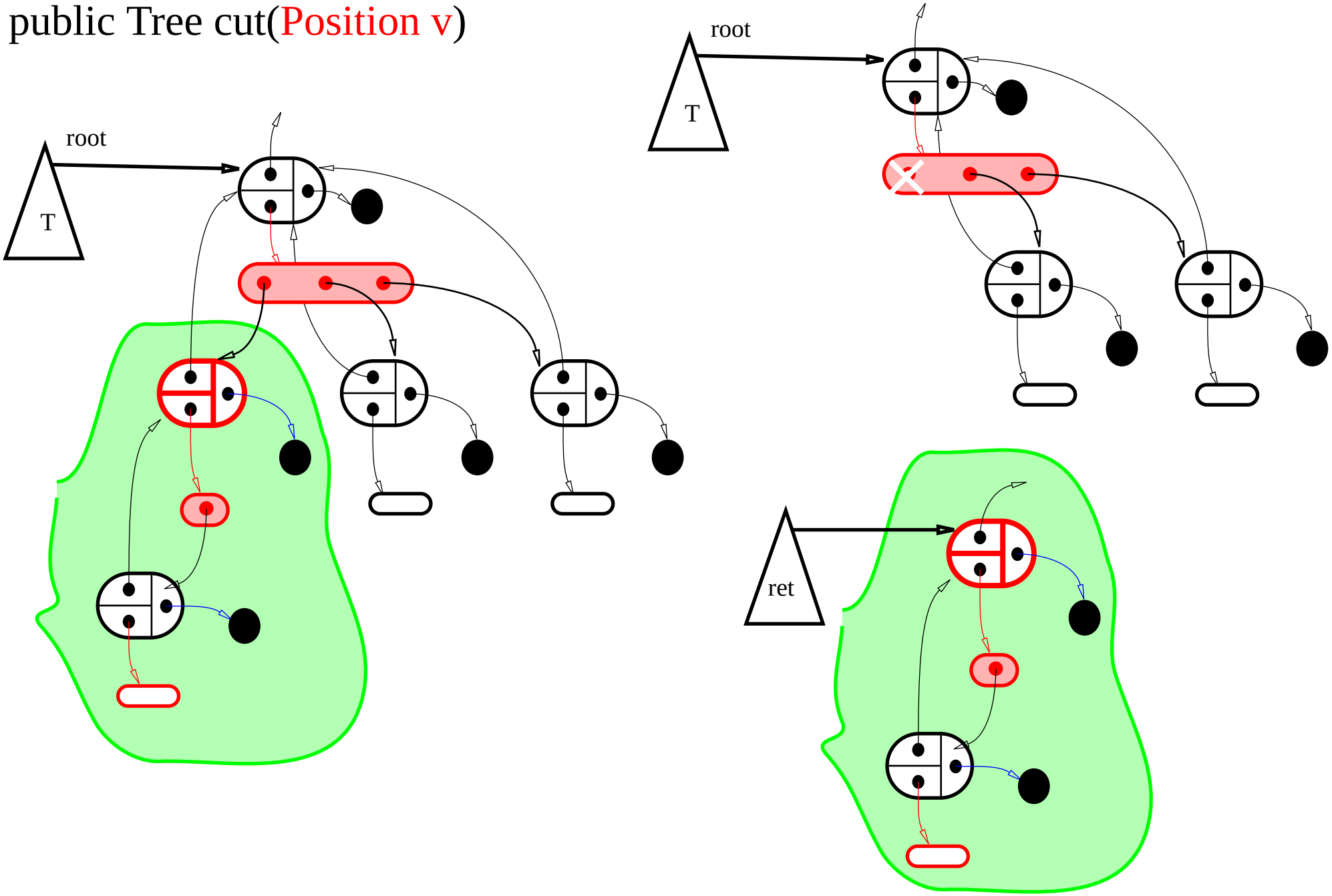
```
public class Node implements Position
{private Object elem;
 private Node parent;
 private Sequence children;

 public Node(Object e, Node p) {
     setElement(e); setParent(p);
     children= new SequenceLL(); }
 public Object element() { return elem; }
 public void setElement(Object e) { elem= e; }

 public Node parent() { return parent; }
 public void setParent(Node p) { parent= p; }
 public PositionIterator children() { return children.positions(); }
 public void addChild(Node c) { . . . }
 public void removeChild(Node c) { . . . }
```

LenketStruktur implementasjon av Tree ADT

```
public class TreeLS implements Tree {
  private Node root; private int size;

  public TreeLS(Node n) {  size= 1; root= n;
    n.setParent(null);  }

  public TreeLS(Object e) {
    this(new Node(e, null)); }

  public Position root() { return root; }

  public Position parent(Position v) throws ... {
    return ok(v).parent(); }

  public PositionIterator children(Position v) throws.. {
    gjør om ok(v).children() til Iterator }
  //ikke ok(v).children().elements();}

  public boolean isInternal(Position v) throws ... {
    return !ok(v).children().isEmpty());}

  public boolean isExternal(Position v) throws ... {
    return ok(v).children().isEmpty());}

  public boolean isRoot(Position v) throws ... {
    return (v == root); }

  public int size() { return size; }
```

```
public Object replaceElement(Position v, Object e) throws..{
     Node p = ok(v);
     Object tmp = p . element();
     p.setElement(e); return tmp;  }


public Tree cut(Position v) throws ... {
   Node n = ok(v);
   Node p = ok(parent(v));
   int less  = n . size();
   p.removeChild(n);
   TreeLS ret = new TreeLS(n);
   size = size – less ;
   return ret; }  // v er ikke lenger Position i dette treet



private Node ok(Position p) throws InvalidPosExc {
   if ( p==null || !(p instanceof Node) )
      throw new InvalidPosExc();
   return (Node)p; }
. . .
```

public Tree cut(Position v)

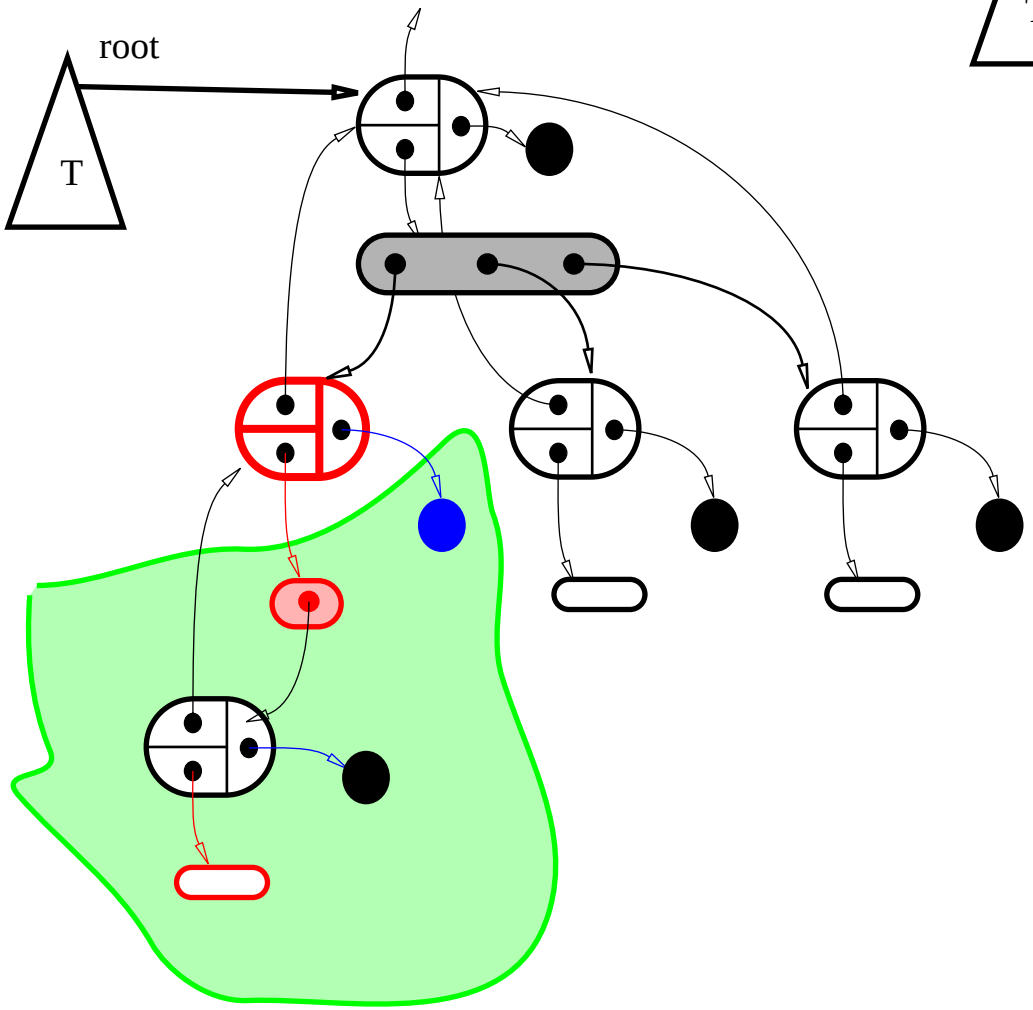


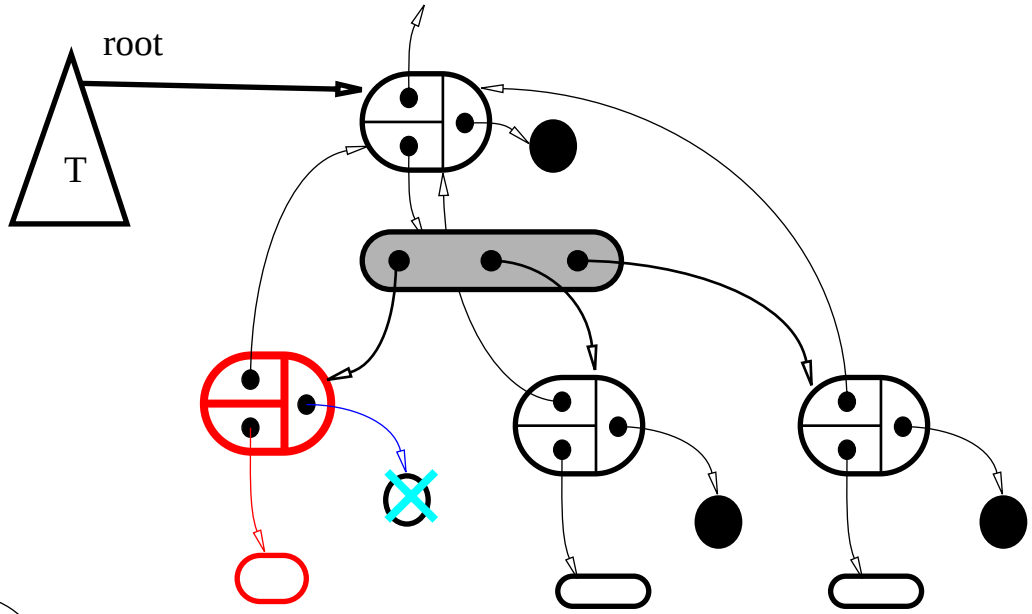
LenketStruktur implementasjon av Tree ADT

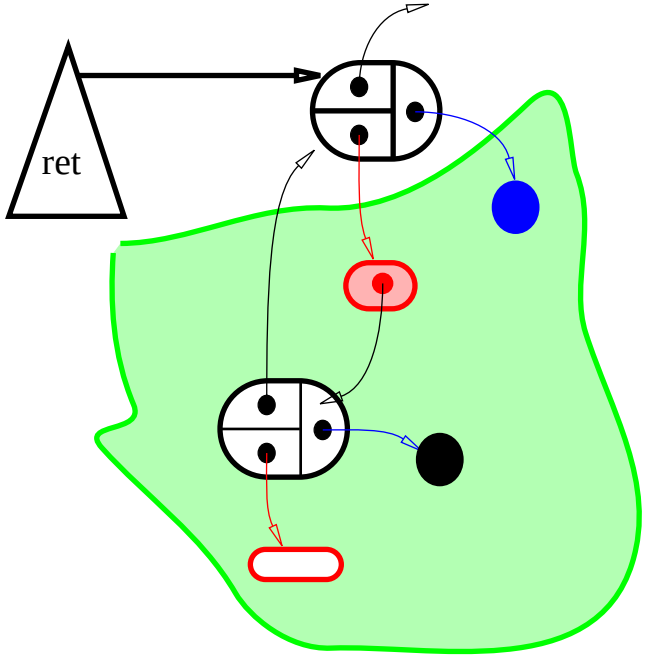
```
public class TreeLS implements Tree {
  private Node root; private int size;

  public TreeLS(Node n) {  size= 1; root= n;
    n.setParent(null);  }

  public TreeLS(Object e) {
    this(new Node(e, null)); }

  public Position root() { return root; }

  public Position parent(Position v) throws ... {
    return ok(v).parent(); }

  public PositionIterator children(Position v) throws.. {
    gjør om ok(v).children() til Iterator }
  //ikke ok(v).children().elements();}

  public boolean isInternal(Position v) throws ... {
    return !ok(v).children().isEmpty());}

  public boolean isExternal(Position v) throws ... {
    return ok(v).children().isEmpty());}

  public boolean isRoot(Position v) throws ... {
    return (v == root); }

  public int size() { return size; }
```

```
  public Object replaceElement(Position v, Object e) throws..{
      Node p = ok(v);
      Object tmp = p . element();
      p.setElement(e); return tmp;  }

  public Tree cut(Position v) throws ... {
    Node n= ok(v);
    TreeLS ret= new TreeLS(n.element());
    Node retRoot = (Node)ret.root();
    PositionIterator ch= children(n);
    while (ch.hasMoreElements()) {
      ((Node)ch.nextElement()).setParent(retRoot); }
    retRoot.setChildren(n.children());
    n.setElement(null);  n.setChildren(null);
    size = size – ret.size() + 1;
    return ret; }  // v er fortsatt Position i dette treet

  private Node ok(Position p) throws InvalidPosExc {
    if ( p==null || !(p instanceof Node) )
       throw new InvalidPosExc();
    return (Node)p; }
  . . .
```

public Tree cut(Position v)



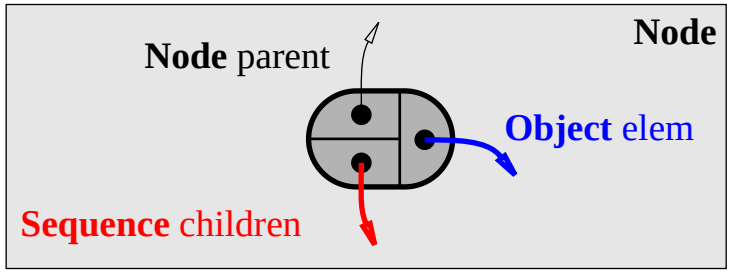




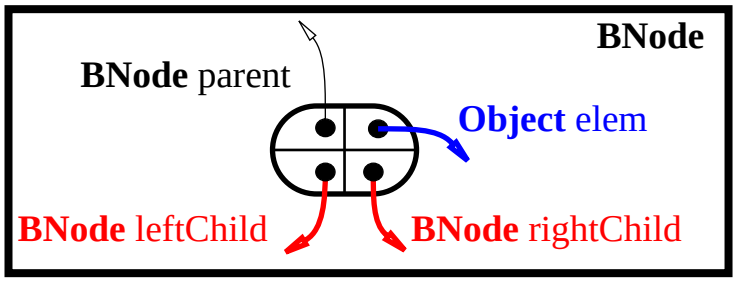
Implementasjon av BinaryTree ADT

I. UTVID KLASSEN TREELS SLIK AT:

- Container children() *alltid har 0 eller 2 elementer – pass på* link(p,T), replaceSubtree(p,T)
- *implementer* Position leftChild(), Position rightChild() – *skill i* children()



II. BRUK EN ANNEN BNODE

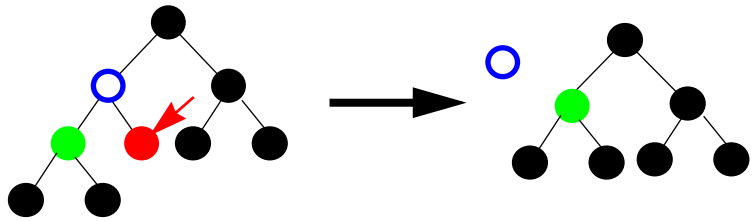



```
public class BNode implements Position
{ private Object elem;
  private BNode parent, left, right;
  public BNode(Object e, BNode p) {
    setElement(e); setParent(p);  }
  public Object element() { return elem; }
  . . .
  public BNode leftChild() { return left; }
  protected void setLeft(BNode p) {
    left = p; p.setParent(this); }
  public BNode rightChild() { return right; }
  protected void setRight(BNode p) {
    right = p; p.setParent(this); ; }
```

LenketStruktur implementasjon av BinaryTree ADT

```
public class BTreeLS implements BinTree {
  private Position root;
  public BTreeLS(Object e) {
    root = new BNode(e, null);  }
  public Position parent(Position v) throws ... {
    return ok(v).parent();   }
  public Position leftChild(Position v) throws ... {
    return ok(v).leftChild();   }
  public Position setLeft(Position v, Object o) throws...{
    BNode n= ok(v);
    if (isExternal(n)) expandExternal(n);
    n.setLeft( new BNode(o, n, this) );
    return n.leftChild();   }
  public Position rightChild(Position v) throws..{ ... }
  public Position setRight(Position v, Object c) throws..{.}
  private BNode ok(Position p) throws InvalidPosExc {
    if ( p==null || !(p instanceof BNode) )
       throw new InvalidPosExc();
    return (BNode)p;    }
  public int size() { return size(root); }
  private int size(Position p) { if (isExternal(p)) return 1;
    else return size(leftChild(p)) + size(rightChild(p)) +1; }
```

```
public boolean isInternal(Position v) throws ... {
  BNode n= ok(v);
  return n.leftChild()!=null || n.rightChild()!=null);  }
public boolean isExternal(Position v) throws ... {
  return ! isInternal(v) ;   }
public void expandExternal(Position v) {
  BNode n= ok(v);
  if (isExternal(n)) {
    n.setLeft( new BNode(null, n) );
    n.setRight( newBNode(null, n) ); }    }
public void removeAboveExternal(Position v) throws...{
  BNode n = ok(v);  // Object o = v.element();
  if ( isInternal(n) || isRoot(n) ) throw ...
  BNode par = ok(parent(n)) ;
  BNode sib = ok(sibling(n));
  par.setElement(sib.element());
  par.setLeft(sib.leftChild());
  par.setRight(sib.rightChild());
}
```



alt unntatt positions(), elements(), size() *er* O(1)

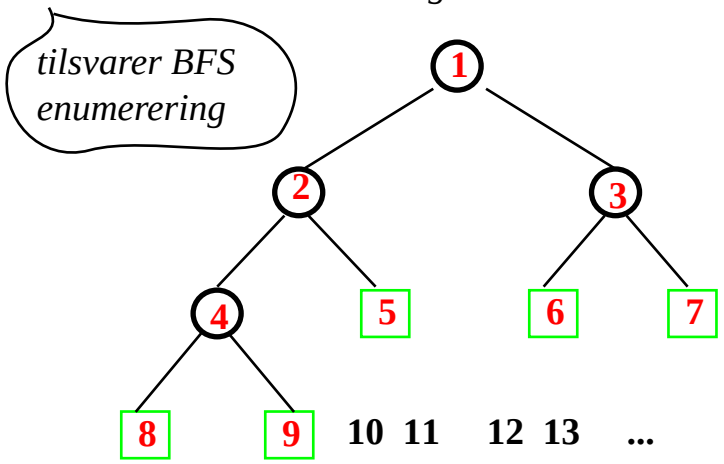
Sequence implementasjon av BinaryTree ADT

I. Data Representasjon

for en Position v i treet T, la sn(v) være et tall gitt ved:

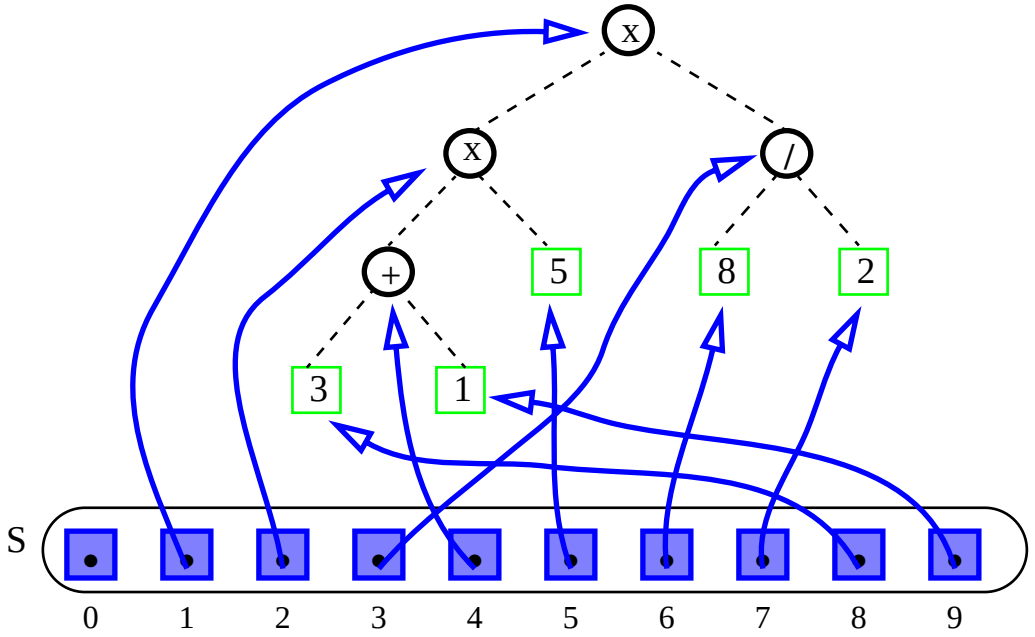
- hvis T.isRoot(v) så sn(v) = 1
- hvis T.leftChild(v) == u så sn(u) = 2*sn(v)
- hvis T.rightChild(v) == u så sn(u) = 2*sn(v) + 1

sn *bestemmer nodens stilling i sekvensen*



II. Data Struktur

Sequence S // *Sekvens-Position er Tree-Position*



Position **leftChild**(Position p) {
return S . atRank(S.rankOf(p)*2);}
Position **root**() { return S . atRank(1); }

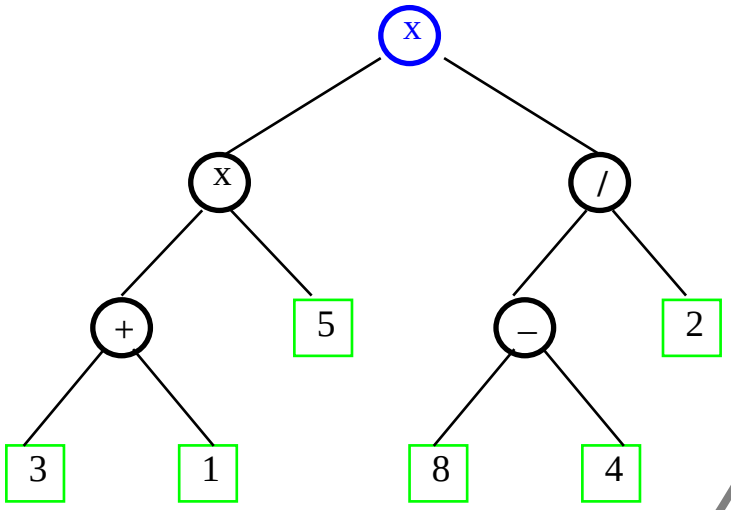
- *alle* BinaryTree *operasjoner (unntatt positions(), elements()) er* **O**(1) ***relativt*** *til* Sequence
- *Spesielt adekvat for "**komplette**" binære trær*

III. DataInvariant :

- S.atRank[1] == root
- S.atRank[2*v] == T.leftChild(S.atRank[v])
- S.atRank[2*v+1] == T.rightChild(S.atRank[v])

6. Syntakstrær

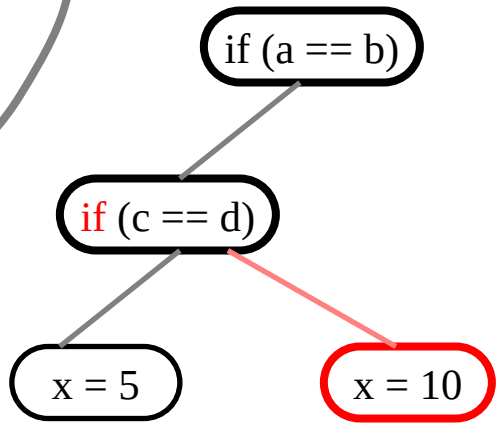
((3 + 1) x 5) x ((8 – 4) / 2)

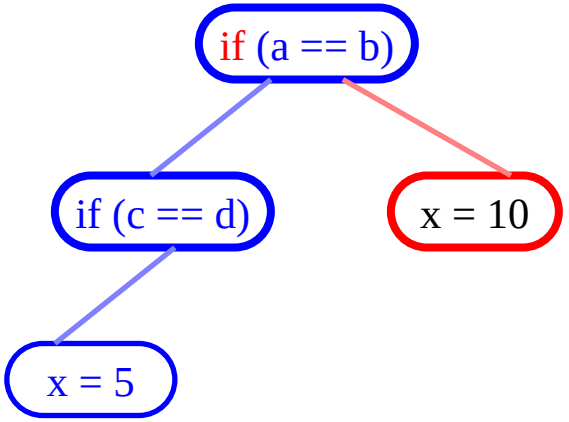



```
if (a == b)
if (c == d) then x = 5;
else x = 10;
```

```
if (a == b) {
    if (c == d)  then x = 5;
    else x = 10;
}
```

```
if (a == b)
    { if (c == d)  then x = 5; }
else x = 10;
```





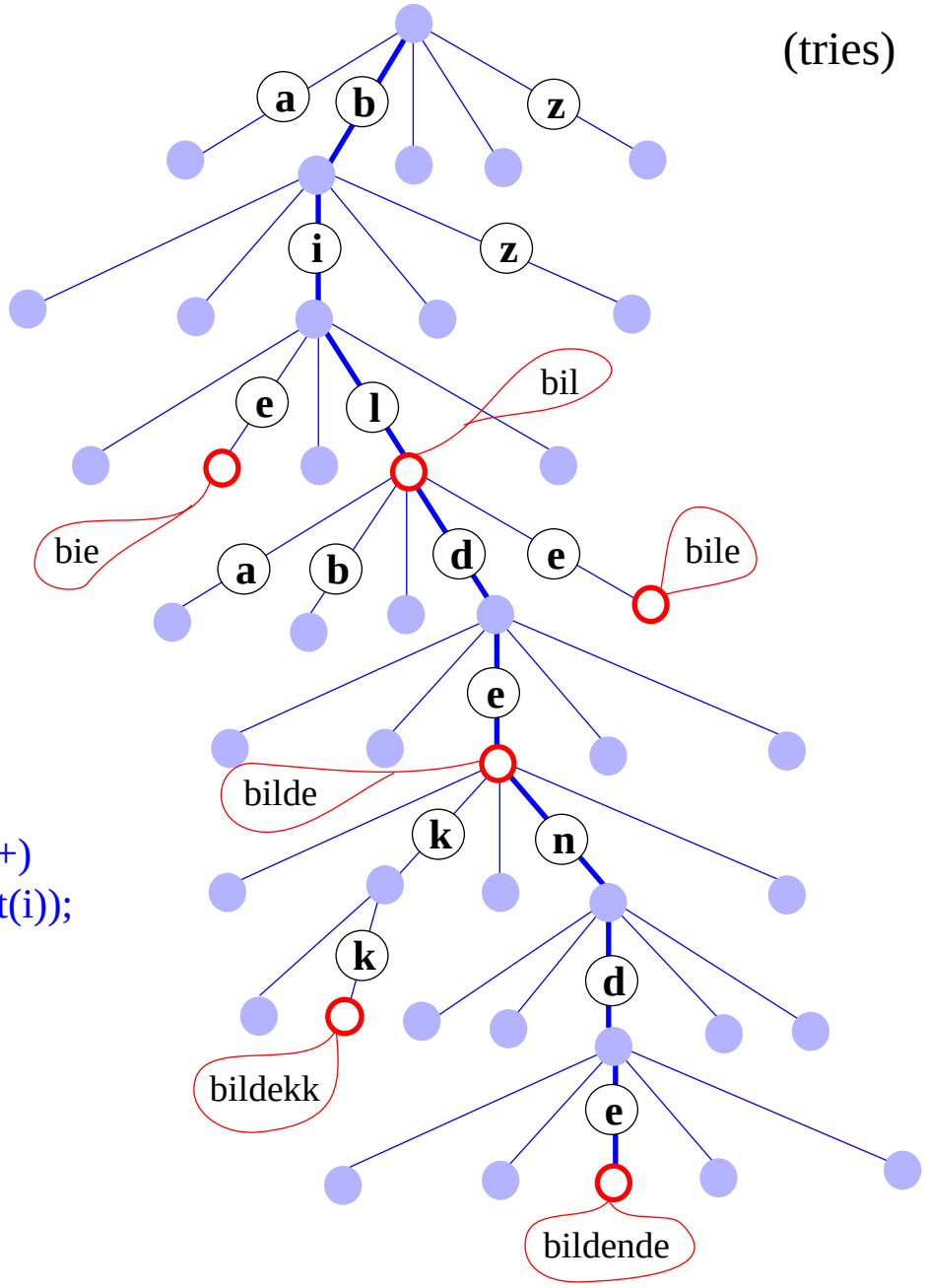
6. Ordbok-søking (totalt ordnede barn)

(tries)

...
bie
bil
bilateral
bilbelte
bilde
bildekk
bildende
bile
....

finn(String X)
= **O**(høyde(treet))
ev. **O**(log n)

```
Position finn(String X)
    p = root;
    for (int i=0; i<X.length;i++)
        p = p.childNo(X.charAt(i));
        if (p == null) break;
    if (p == null || !p.isRed())
        finnes-ikke
    else return p;

    = O(X.length)
```



Oppsummering

1. Trær og Binære Trær:

- *definisjoner og terminologi*
- *egenskaper*

2. Tre-algoritmer – traversering:

- *DFS og BFS*
- *DFS :*
  - *– pre– og postorder,*
  - *– inorder for BinaryTree*

3. Tree og BinaryTree ADT

4. Implementasjon av trær:

- *LenketStruktur*
- *Sekvens – BinaryTree*
- *kompleksitet*