STRINGS AND PATTERN MATCHING

- Brute Force,Rabin-Karp, Knuth-Morris-Pratt
- Regular Expressions

String Searching

- The previous slide is not a great example of what is meant by "String Searching." Nor is it meant to ridicule people without eyes....
- The object of string searching is to find the location of a specific text pattern within a larger body of text (e.g., a sentence, a paragraph, a book, etc.).
- As with most algorithms, the main considerations for string searching are speed and efficiency.
- There are a number of string searching algorithms in existence today, but the three we shall review are Brute Force,Rabin-Karp, and Knuth-Morris-Pratt.

Brute Force

- The Brute Force algorithm compares the pattern to the text, one character at a time, until unmatching characters are found:

```
TWO ROADS DIVERGED IN A YELLOW WOOD
ROADS
```
```
TWO ROADS DIVERGED IN A YELLOW WOOD
 ROADS
```
```
TWO ROADS DIVERGED IN A YELLOW WOOD
  ROADS
```
```
TWO ROADS DIVERGED IN A YELLOW WOOD
   ROADS
```
```
TWO ROADS DIVERGED IN A YELLOW WOOD
    ROADS
```

  - Compared characters are italicized.
  - Correct matches are in boldface type.

- The algorithm can be designed to stop on either the first occurrence of the pattern, or upon reaching the end of the text.

Brute Force Pseudo-Code

- Here's the pseudo-code

```
do
  if (text letter == pattern letter)
    compare next letter of pattern to next
    letter of text
  else
    move pattern down text by one letter
while (entire pattern found or end of text)
```

```
cool cat Rolo went over the fence
cat
```

```
cool cat Rolo went over the fence
 cat
```

```
cool cat Rolo went over the fence
  cat
```

```
cool cat Rolo went over the fence
   cat
```

```
cool_cat Rolo went over the fence
    cat
```

```
cool cat Rolo went over the fence
     cat
```

Brute Force-Complexity

- Given a pattern M characters in length, and a text N characters in length...
- **Worst case**: compares pattern to each substring of text of length M. For example, M=5.
- This kind of case can occur for image data.

1) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
2) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
3) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
4) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
5) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAH **5 comparisons made**
....
N) AAAAAAAAAAAAAAAAAAAAAAAAA*AAAAH*
5 comparisons made *AAAAH*

- Total number of comparisons: M (N-M+1)
- Worst case time complexity: O(MN)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- **Best case if pattern found**: Finds pattern in first M positions of text. For example, M=5.

1) *AAAAA*AAAAAAAAAAAAAAAAAAAAAAAH
AAAAA **5 comparisons made**

- Total number of comparisons: M
- Best case time complexity: O(M)

Brute Force-Complexity(cont.)

- Given a pattern M characters in length, and a text N characters in length...
- **Best case if pattern not found**: Always mismatch on first character. For example, M=5.

1) *A*AAAAAAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
2) A*A*AAAAAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
3) AA*A*AAAAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
4) AAA*A*AAAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**
5) AAAA*A*AAAAAAAAAAAAAAAAAAAAAAAH
   *O*OOOH **1 comparison made**

...

N) AAAAAAAAAAAAAAAAAAAAAAAAAA*A*AAAH
   1 comparison made *O*OOOH

- Total number of comparisons: N
- Best case time complexity: O(N)

Rabin-Karp

- The Rabin-Karp string searching algorithm calculates a **hash value** for the pattern, and for each M-character subsequence of text to be compared.
- If the hash values are unequal, the algorithm will calculate the hash value for next M-character sequence.
- If the hash values are equal, the algorithm will do a Brute Force comparison between the pattern and the M-character sequence.
- In this way, there is only one comparison per text subsequence, and Brute Force is only needed when hash values match.
- Perhaps an example will clarify some things...

Rabin-Karp Example

Hash value of "AAAAA" is 37

Hash value of "AAAAH" is 100

1) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
AAAAH
37≠100 **1 comparison made**

2) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
AAAAH
37≠100 **1 comparison made**

3) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
AAAAH
37≠100 **1 comparison made**

4) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
AAAAH
37≠100 **1 comparison made**

...

N) AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAH
AAAAH
5 comparisons made 100=100

Rabin-Karp Algorithm

pattern is M characters long

hash_p=hash value of pattern
hash_t=hash value of first M letters in
body of text

```
do
  if (hash_p == hash_t)
    brute force comparison of pattern
    and selected section of text
  hash_t = hash value of next section of
           text, one character over
while (end of text or
       brute force comparison == true)
```

Rabin-Karp

- Common Rabin-Karp questions:

  "What is the hash function used to calculate values for character sequences?"

  "Isn't it time consuming to hash every one of the M-character sequences in the text body?"

  "Is this going to be on the final?"

- To answer some of these questions, we'll have to get mathematical.

Rabin-Karp Math

- Consider an M-character sequence as an M-digit number in base $\boldsymbol{b}$, where $\boldsymbol{b}$ is the number of letters in the alphabet. The text subsequence t[i .. i+M-1] is mapped to the number

$$\boldsymbol{x}(i) = \boldsymbol{t}[i]\cdot\boldsymbol{b}^{M-1} + \boldsymbol{t}[i+1]\cdot\boldsymbol{b}^{M-2} + \ldots + \boldsymbol{t}[i+M-1]$$

- Furthermore, given x(i) we can compute x(i+1) for the next subsequence t[i+1 .. i+M] in constant time, as follows:

$$\boldsymbol{x}(i+1) = \boldsymbol{t}[i+1]\cdot\boldsymbol{b}^{M-1} + \boldsymbol{t}[i+2]\cdot\boldsymbol{b}^{M-2} + \ldots + \boldsymbol{t}[i+M]$$

$\boldsymbol{x}(i+1) = \boldsymbol{x}(i)\cdot\boldsymbol{b}$	Shift left one digit
$- \boldsymbol{t}[i]\cdot\boldsymbol{b}^{M}$	Subtract leftmost digit
$+ \boldsymbol{t}[i+M]$	Add new rightmost digit

- In this way, we never explicitly compute a new value. We simply adjust the existing value as we move over one character.

Rabin-Karp Math Example

- Let's say that our alphabet consists of 10 letters.
- our alphabet = a, b, c, d, e, f, g, h, i, j
- Let's say that "a" corresponds to 1, "b" corresponds to 2 and so on.

The hash value for string "cah" would be ...

$$3*100 + 1*10 + 8*1 = 318$$

Rabin-Karp Mods

- If M is large, then the resulting value (~bM) will be enormous. For this reason, we hash the value by taking it **mod** a prime number ***q***.
- The **mod** function (% in Java) is particularly useful in this case due to several of its inherent properties:
  - [(x mod q) + (y mod q)] mod q = (x+y) mod q
  - (x mod q) mod q = x mod q
- For these reasons:

$$\boldsymbol{h}(i) = ((\boldsymbol{t}[i]\cdot \boldsymbol{b}^{M-1} \bmod \boldsymbol{q}) + (\boldsymbol{t}[i+1]\cdot \boldsymbol{b}^{M-2} \bmod \boldsymbol{q}) + \ldots + (\boldsymbol{t}[i+M-1] \bmod \boldsymbol{q})) \bmod \boldsymbol{q}$$

$\boldsymbol{h}(i+1) = (\ \boldsymbol{h}(i)\cdot \boldsymbol{b} \bmod \boldsymbol{q}$

Shift left one digit

$-\boldsymbol{t}[i]\cdot \boldsymbol{b}^{M} \bmod \boldsymbol{q}$

Subtract leftmost digit

$+\boldsymbol{t}[i+M] \bmod \boldsymbol{q}\ )$

Add new rightmost digit

$\bmod \boldsymbol{q}$

Rabin-Karp Complexity

- If a sufficiently large prime number is used for the *hash function*, the hashed values of two different patterns will usually be distinct.
- If this is the case, searching takes O(N) time, where N is the number of characters in the larger body of text.
- It is always possible to construct a scenario with a worst case complexity of O(MN). This, however, is likely to happen only if the prime number used for hashing is small.

The Knuth-Morris-Pratt Algorithm

- The Knuth-Morris-Pratt (KMP) string searching algorithm differs from the brute-force algorithm by keeping track of information gained from previous comparisons.
- A failure function (*f*) is computed that indicates how much of the last comparison can be reused if it fails.
- Specifically, *f* is defined to be the longest prefix of the pattern P[0,..,j] that is also a suffix of P[1,..,j]
  - Note: **not** a suffix of P[0,..,j]
- Example:
  - value of the KMP failure function:

j	0	1	2	3	4	5
P[*j*]	a	b	a	b	a	c
f(*j*)	0	0	1	2	3	0

- This shows how much of the beginning of the string matches up to the portion immediately preceding a failed comparison.
  - if the comparison fails at (4), we know the a,b in positions 2,3 is identical to positions 0,1

The KMP Algorithm (contd.)

- the KMP string matching algorithm: Pseudo-Code

Algorithm KMPMatch(T,P)
  Input: Strings T (text) with n characters and P (pattern) with m characters.
  Output: Starting index of the first substring of T matching P, or an indication that P is not a substring of T.

```
f ← KMPFailureFunction(P) {build failure function}
i ← 0
j ← 0
while i < n do
   if P[j] = T[i] then
      if j = m - 1 then
         return i - m - 1 {a match}
      i ← i + 1
      j ← j + 1
   else if j > 0 then {no match, but we have advanced}
      j ← f(j-1) {j indexes just after matching prefix in P}
   else
      i ← i + 1
return "There is no substring of T matching P"
```

The KMP Algorithm (contd.)

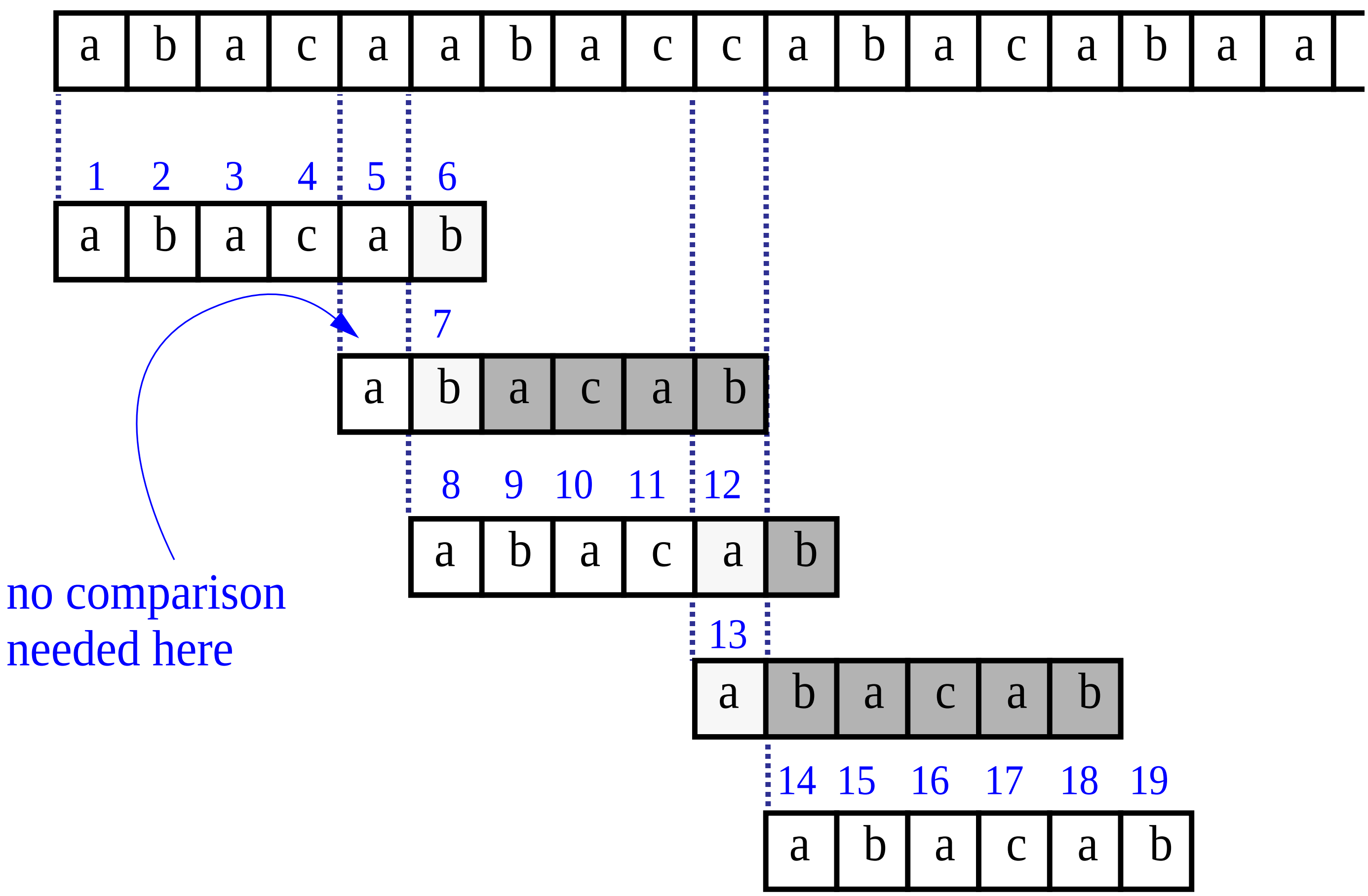
- The KMP failure function: Pseudo-Code

Algorithm KMPFailureFunction(P);
  Input: String P (pattern) with m characters
  Ouput: The faliure function f for P, which maps j to the length of the longest prefix of P that is a suffix of $P[1,..,j]$

```
i ← 1
j ← 0
while i ≤ m-1 do
   if P[j] = T[j] then
      {we have matched j + 1 characters}
      f(i) ← j + 1
      i ← i + 1
      j ← j + 1
   else if j > 0 then
      {j indexes just after a prefix of P that matches}
      j ← f(j-1)
   else
      {there is no match}
      f(i) ← 0
      i ← i + 1
```

The KMP Algorithm (contd.)

- A graphical representation of the KMP string searching algorithm



The KMP Algorithm (contd.)

- Time Complexity Analysis
- define $k = i - j$
- In every iteration through the while loop, one of three things happens.
  - 1) if $T[i] = P[j]$, then i increases by 1, as does j k remains the same.
  - 2) if $T[i] \mathrel{!}= P[j]$ and $j > 0$, then i does not change and k increases by at least 1, since k changes from $i - j$ to $i - f(j-1)$
  - 3) if $T[i] \mathrel{!}= P[j]$ and $j = 0$, then i increases by 1 and k increases by 1 since j remains the same.
- Thus, each time through the loop, either i or k increases by at least 1, so the greatest possible number of loops is $2n$
- This of course assumes that f has already been computed.
- However, f is computed in much the same manner as KMPMatch so the time complexity argument is analogous. KMPFailureFunction is $\boldsymbol{O}(m)$
- Total Time Complexity: $\boldsymbol{O}(n + m)$

Regular Expressions

- notation for describing a set of strings, possibly of infinite size
- ε denotes the empty string
- **ab + c** denotes the set {ab, c}
- **a*** denotes the set {ε, a, aa, aaa, ...}
- Examples
  - **(a+b)*** all the strings from the alphabet {a,b}
  - **b*(ab*a)*b*** strings with an even number of a's
  - **(a+b)*sun(a+b)*** strings containing the pattern "sun"
  - **(a+b)(a+b)(a+b)a** 4-letter strings ending in a