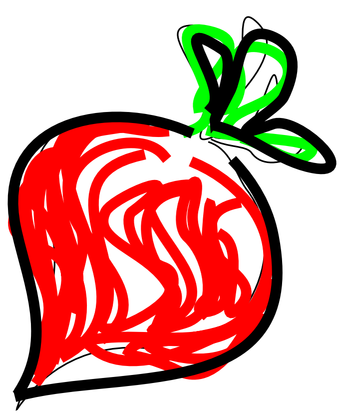
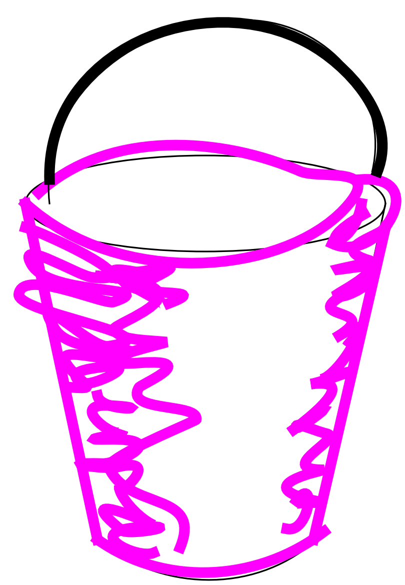
More Sorting

- radix sort
- bucket sort
- in-place sorting
- how fast can we sort?





Radix Sort

- Unlike other sorting methods, radix sort considers the structure of the keys

- Assume keys are represented in a base M number system (M = radix), i.e., if M = 2, the keys are represented in binary

	8	4	2	1	weight
9 =	1	0	0	1	($b = 4$)
	3	2	1	0	bit #

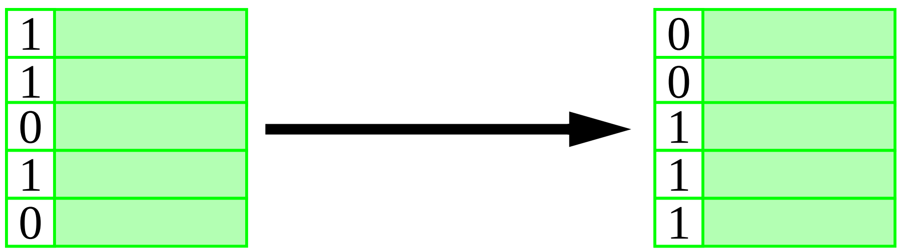
- Sorting is done by comparing bits in the same position

- Extension to keys that are alphanumeric strings

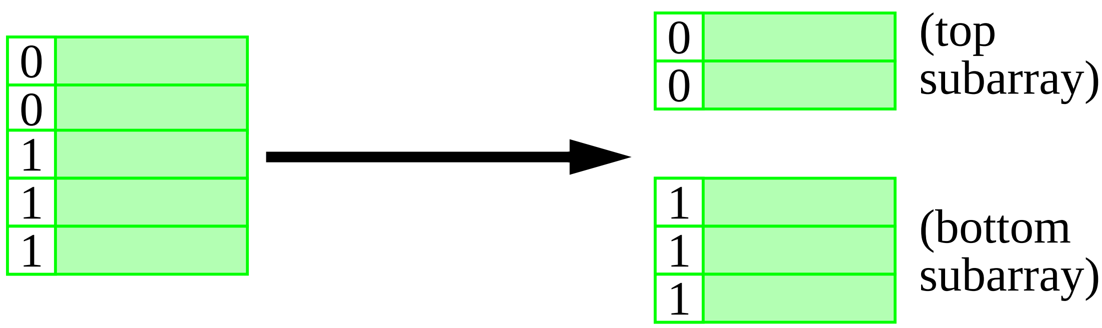
Radix Exchange Sort

Examine bits from ***left*** to ***right***:

1. Sort array with respect to leftmost bit



2. Partition array



3. <u>Recursion</u>

- recursively sort top subarray, ignoring leftmost bit
- recursively sort bottom subarray, ignoring leftmost bit

Time to sort n b-bit numbers: $O(b\ n)$

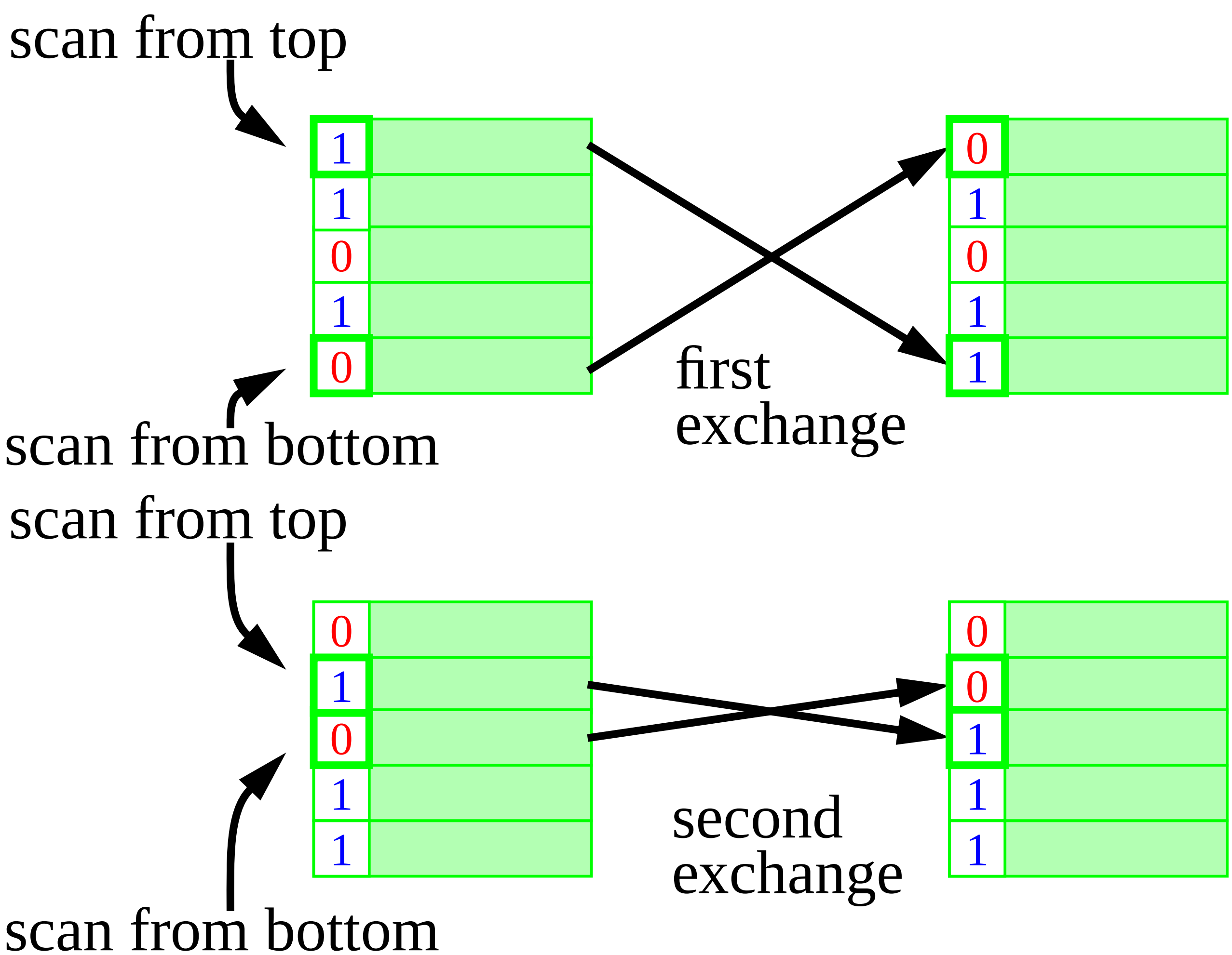
Radix Exchange Sort

How do we do the sort from the previous page?
Same idea as partition in Quicksort.

repeat

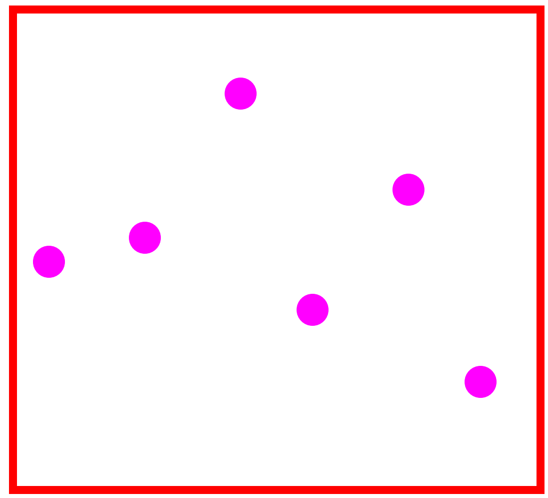
scan top-down to find key starting with 1;
scan bottom-up to find key starting with 0;
exchange keys;

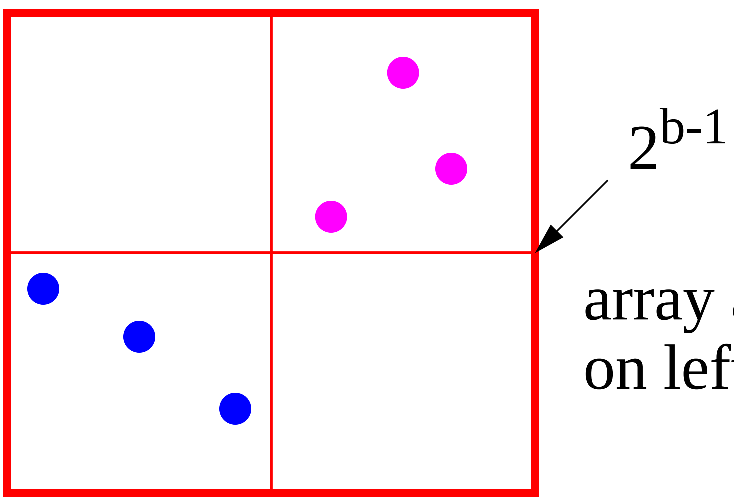
until scan indices cross;



Radix Exchange Sort

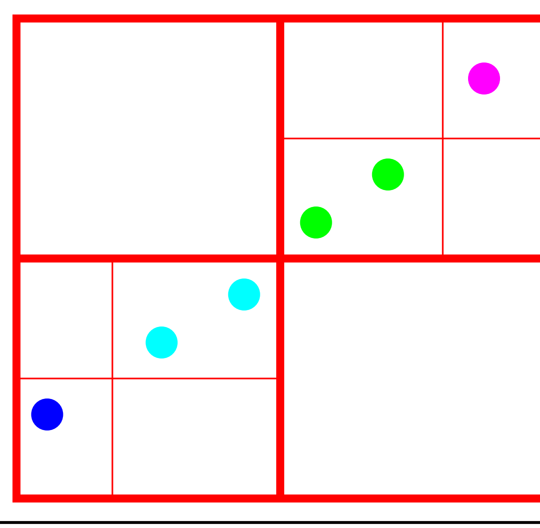
array before sort





array after sort
on leftmost bit

array after recursive
sort on second from
leftmost bit



Radix Exchange Sort vs. Quicksort

Similarities

- both partition array
- both recursively sort sub-arrays

Differences

- ***Method of partitioning***
  - radix exchange divides array based on greater than or less than 2^{b-1}
  - quicksort partitions based on greater than or less than some element of the array
- ***Time complexity***
  - Radix exchange O (bn)
  - Quicksort average case O ($n \log n$)

Straight Radix Sort

Examines bits from ***right*** to ***left***

for k := 0 **to** b–1
sort the array in a ***stable*** way,
looking only at bit k

						First, sort these			Next, sort these digits			Last, sort these.		
0	1	0		0	1	0		0	0	0		0	0	0
0	0	0		0	0	0		1	0	0		0	0	1
1	0	1		1	0	0		1	0	1		0	1	0
0	0	1		1	1	0		0	0	1		0	1	1
1	1	1		1	0	1		0	1	0		1	0	0
0	1	1		0	0	1		1	1	0		1	0	1
1	0	0		1	1	1		1	1	1		1	1	0
1	1	0		0	1	1		0	1	1		1	1	1

Note order of these bits after sort.

I forgot what it means to “sort in a stable way”!!!

In a stable sort, the initial relative order of equal keys is unchanged.

For example, observe the first step of the sort from the previous page:

0	1	0
0	0	0
1	0	1
0	0	1
1	1	1
0	1	1
1	0	0
1	1	0

$\longrightarrow$

0	1	0
0	0	0
1	0	0
1	1	0
1	0	1
0	0	1
1	1	1
0	1	1

Note that the relative order of those keys ending with 0 is unchanged, and the same is true for elements ending in 1

The Algorithm is Correct (right?)

- We show that any two keys are in the correct relative order at the end of the algorithm

- Given two keys, let ***k*** be the leftmost bit-position where they differ

0	1	0	1	1

0	1	1	0	1

k

- At step ***k*** the two keys are put in the correct relative order

- Because of ***stability***, the successive steps do not change the relative order of the two keys

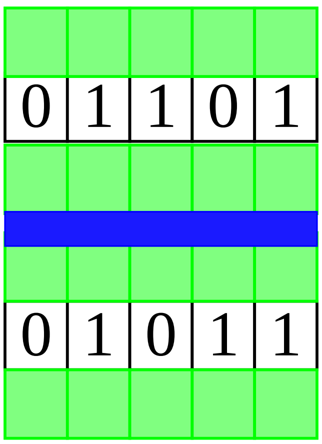
For Instance,

Consider a sort on an array with these two keys:

0	1	0	1	1

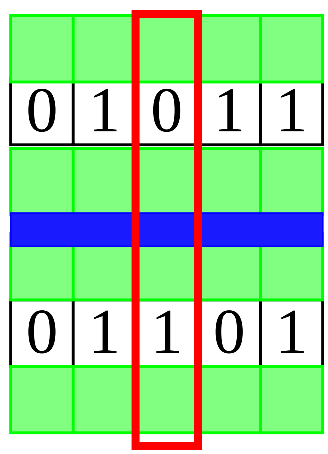
0	1	1	0	1

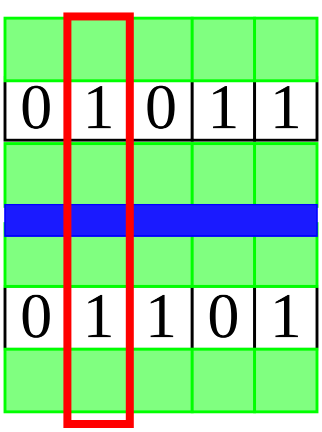
k



It makes no difference what order they are in when the sort begins.

When the sort visits bit ***k*,** the keys are put in the correct relative order.





Because the sort is stable, the order of the two keys will not be changed when bits > ***k*** are compared.

Radix sorting can be applied to decimal numbers

First, sort these digits

Next, sort these digits

Last, sort these.

0	3	2
2	2	4
0	1	6
0	1	5
0	3	1
1	6	9
1	2	3
2	5	2

0	3	1
0	3	2
2	5	2
1	2	3
2	2	4
0	1	5
0	1	6
1	6	9

0	1	5
0	1	6
1	2	3
2	2	4
0	3	1
0	3	2
2	5	2
1	6	9

0	1	5
0	1	6
0	3	1
0	3	2
1	2	3
1	6	9
2	2	4
2	5	2

Note order of these bits after sort.

Voila!

Straight Radix Sort

Time Complexity

for $k = 0$ **to** $b - 1$
sort the array in a *stable* way,
looking only at bit k

Suppose we can perform the stable sort above in O(n) time. The total time complexity would be

$$O(bn)$$

As you might have guessed, we can perform a stable sort based on the keys' $\boldsymbol{k}^{\text{th}}$ digit in O(n) time.

The method, you ask? Why it's Bucket Sort, of course.

Bucket Sort

- n numbers
- Each number $\in$ {1, 2, 3, ... m}
- Stable
- Time: O ($n + m$)

For example, $m = 3$ and our array is:

2	1	3	1	2

(note that there are two “2”s and two “1”s)

First, we create M “buckets”

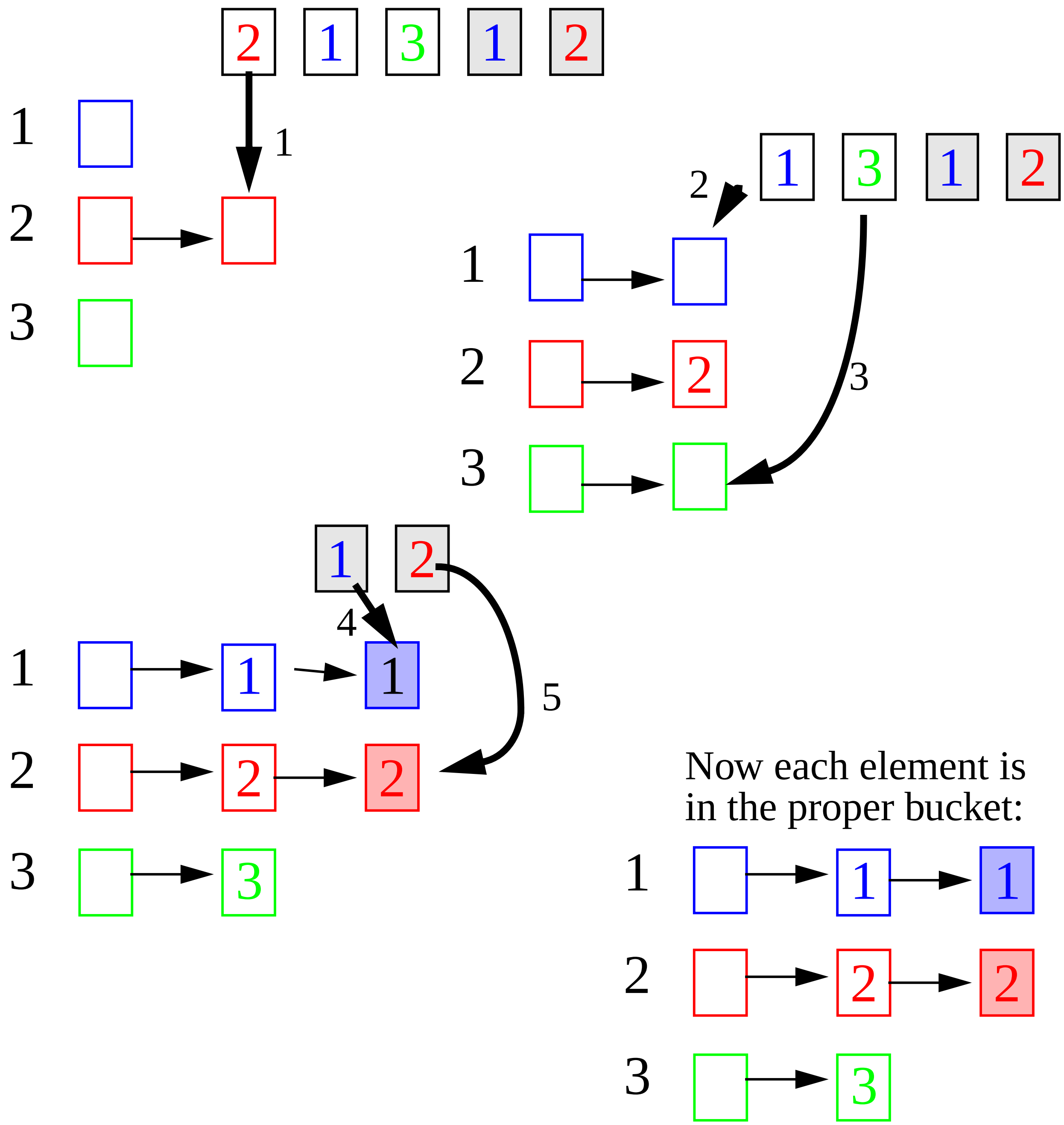
1

2

$m =$ 3

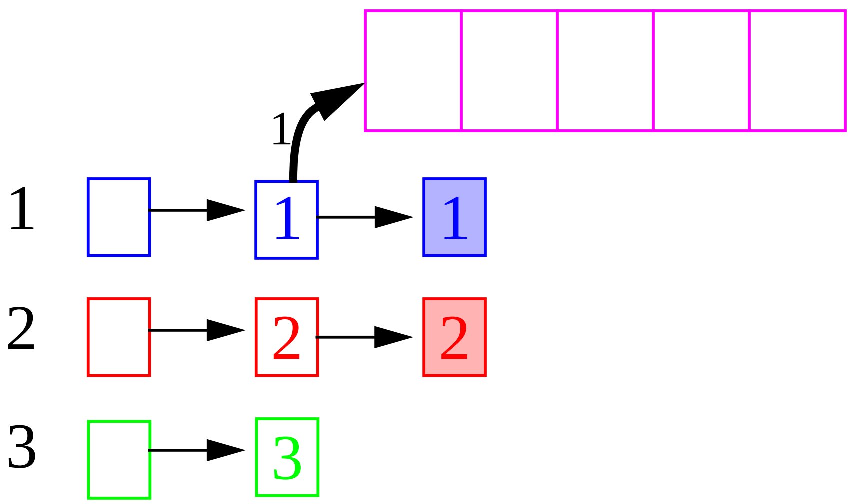
Bucket Sort

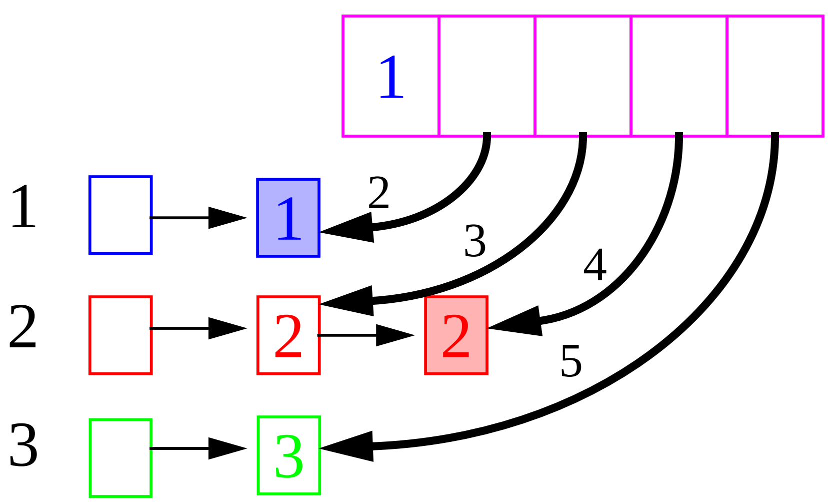
Each element of the array is put in one of the *m* “buckets”



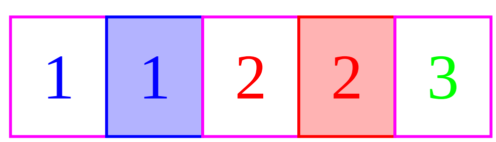
Bucket Sort

Now, pull the elements from the buckets into the array





At last, the sorted array (sorted in a ***stable*** way):



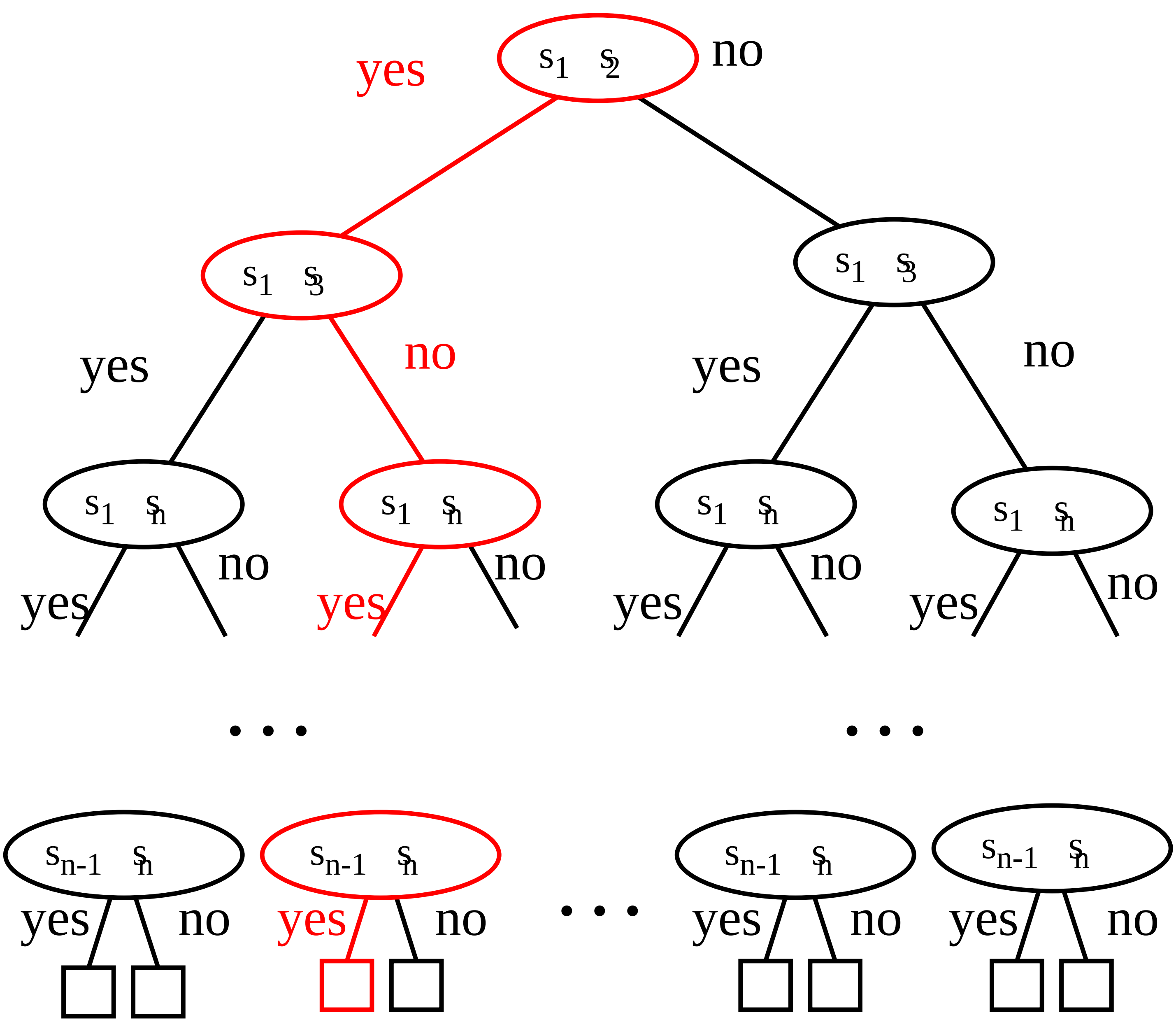
In-Place Sorting

- A sorting algorithm is said to be ***in-place*** if
  - it uses no auxiliary data structures (however, O(1) auxiliary variables are allowed)
  - it updates the input sequence only by means of operations replaceElement and swapElements
- Which sorting algorithms seen so far can be made to work in place?

bubble-sort	Y
selection-sort	
insertion-sort	
heap-sort	
merge-sort	
quick-sort	
radix-sort	
bucket-sort	

Decision Tree for Comparison Based Sorting

- internal node: comparison
- external node: permutation
- algorithm execution: root-to-leaf path



How Fast Can We Sort?

- **Proposition**: The running time of any comparison-based algorithm for sorting an n-element sequence S is $\Omega(n \log n)$.
- **Justification**:
- The running time of a comparison-based sorting algorithm must be equal to or greater than the depth of the decision tree T associated with this algorithm.
- Each internal node of T is associated with a comparison that establishes the ordering of two elements of S.
- Each external node of T represents a distinct permutation of the elements of S.
- Hence T must have at least $n!$ external nodes which again implies T has a height of at least $\log(n!)$
- Since $n!$ has at least $n/2$ terms that are greater than or equal to $n/2$, we have:

$$\log(n!) \quad (n/2)\log(n/2)$$

- Total Time Complexity: $\Omega(n \log n)$.