(2,4) Trees

- Search Trees (but not binary)

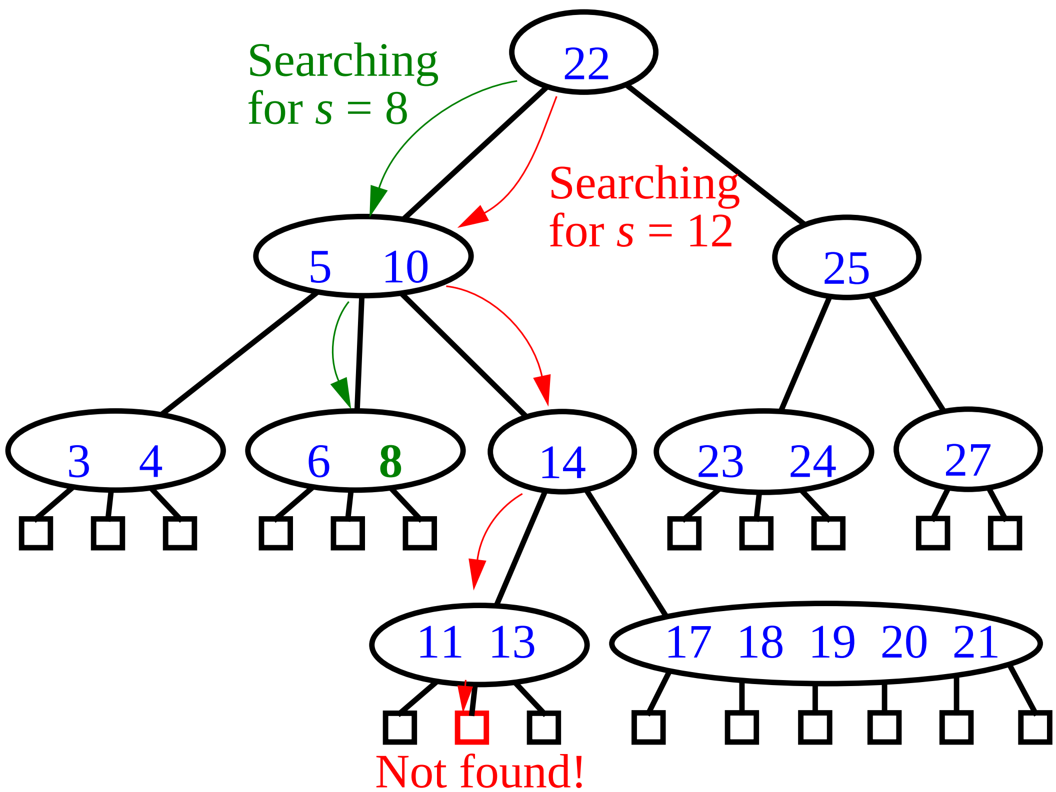
- also known as 2-4, 2-3-4 trees

Multi-way Search Trees

- Each internal node of a multi-way search tree *T*:
  - has at least two children
  - stores a collection of items of the form (*k*, *x*), where *k* is a key and *x* is an element
  - contains *d* - 1 items, where *d* is the number of children
  - "contains" 2 pseudo-items: $k_0 = -\infty$, $k_d = \infty$
- Children of each internal node are "between" items
  - all keys in the subtree rooted at the child fall between keys of those items
- External nodes are just placeholders

Multi-way Searching

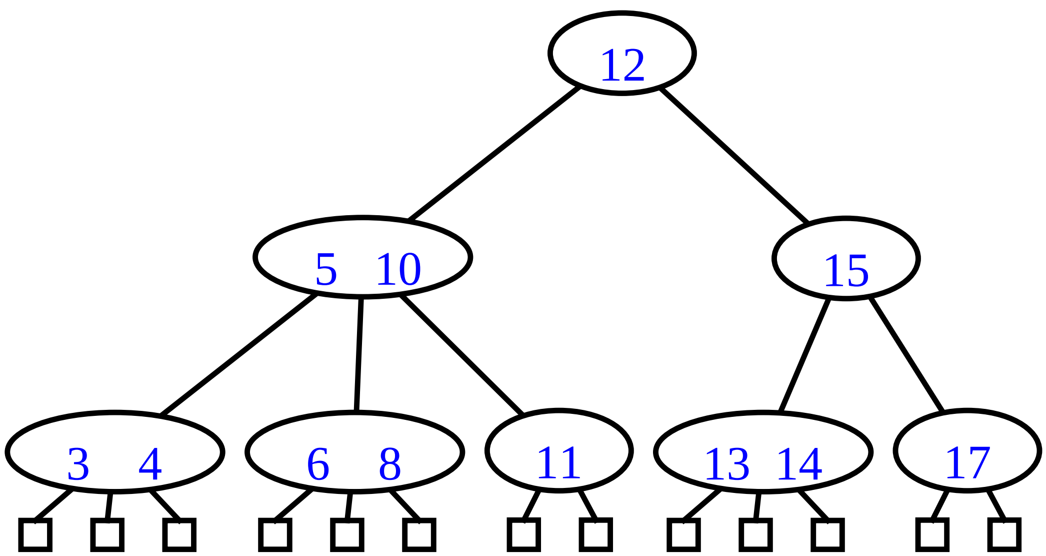
- Similar to binary searching
- If search key $s < k_1$, search the leftmost child
- If $s > k_{d-1}$, search the rightmost child
- That's it in a binary tree; what about if $d > 2$?
- Find two keys k_{i-1} and k_i between which s falls, and search the child v_i.



- What would an in-order traversal look like?

(2,4) Trees

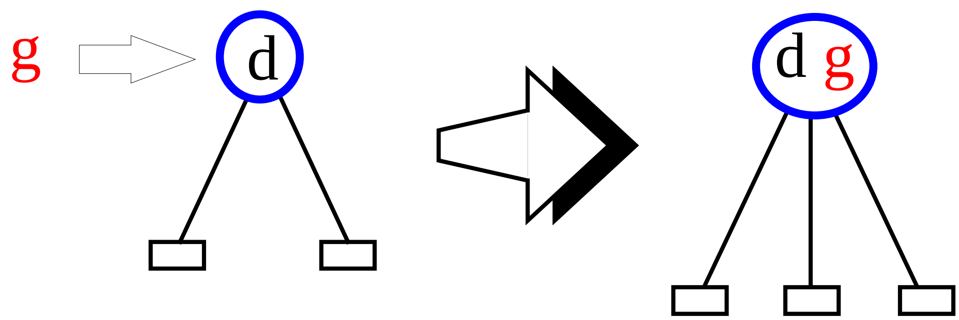
- At most 4 children
- All external nodes have same depth
- Height h of (2,4) tree is $O(\log n)$.
- How is this fact useful in searching?



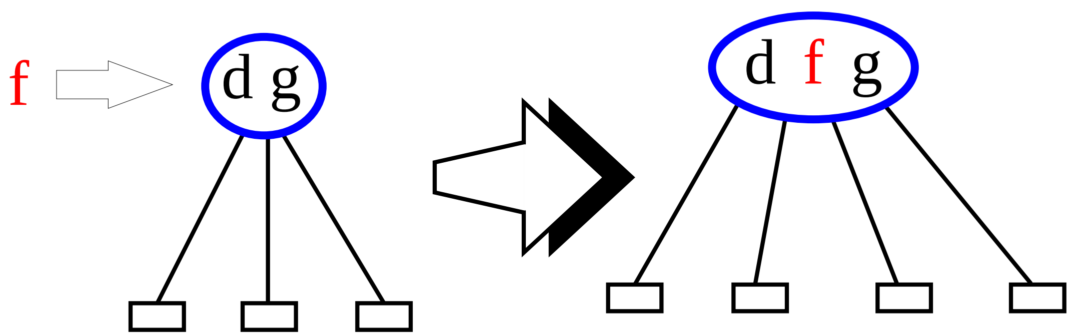
Insertion into (2,4) Trees

- Insert the new key at the lowest internal node reached in the search

  - *2-node* becomes *3-node*



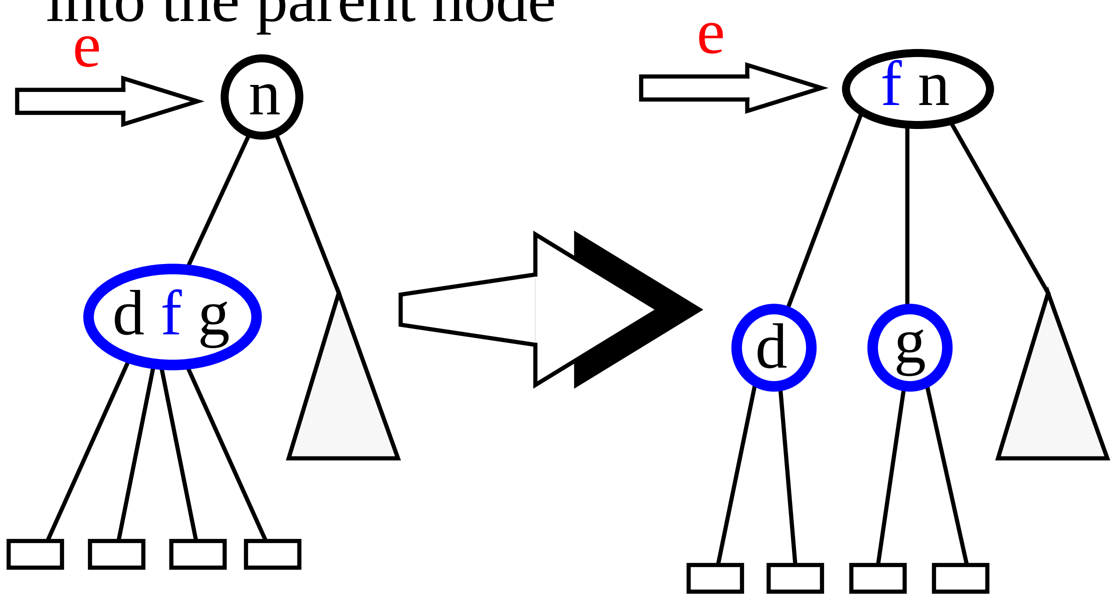
  - *3-node* becomes *4-node*

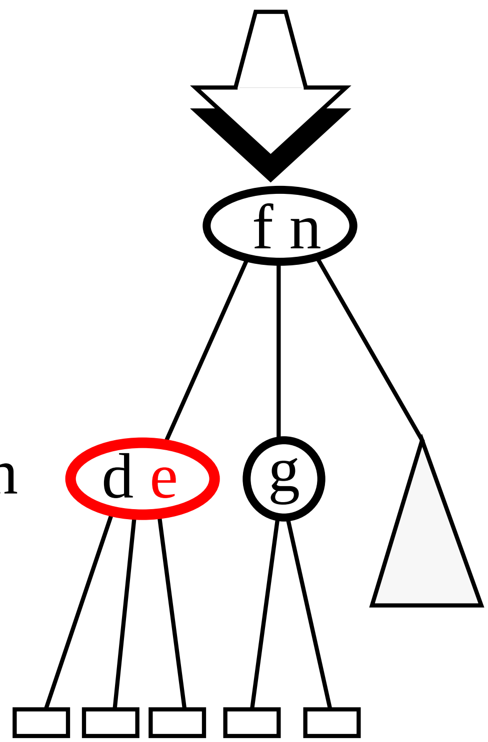


- What about a *4-node*?
  - We can't insert another key!

Top Down Insertion

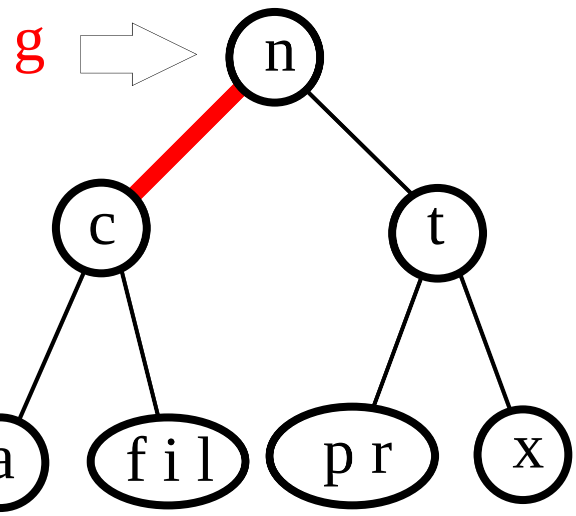
- In our way down the tree, whenever we reach a ***4-node***, we break it up into two ***2-nodes***, and move the middle element up into the parent node

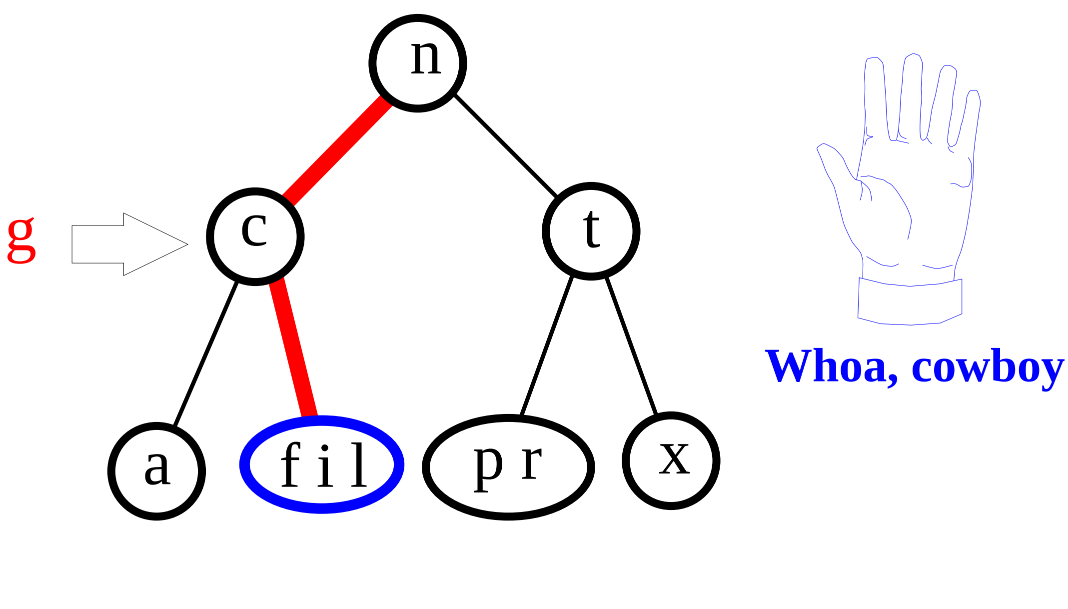


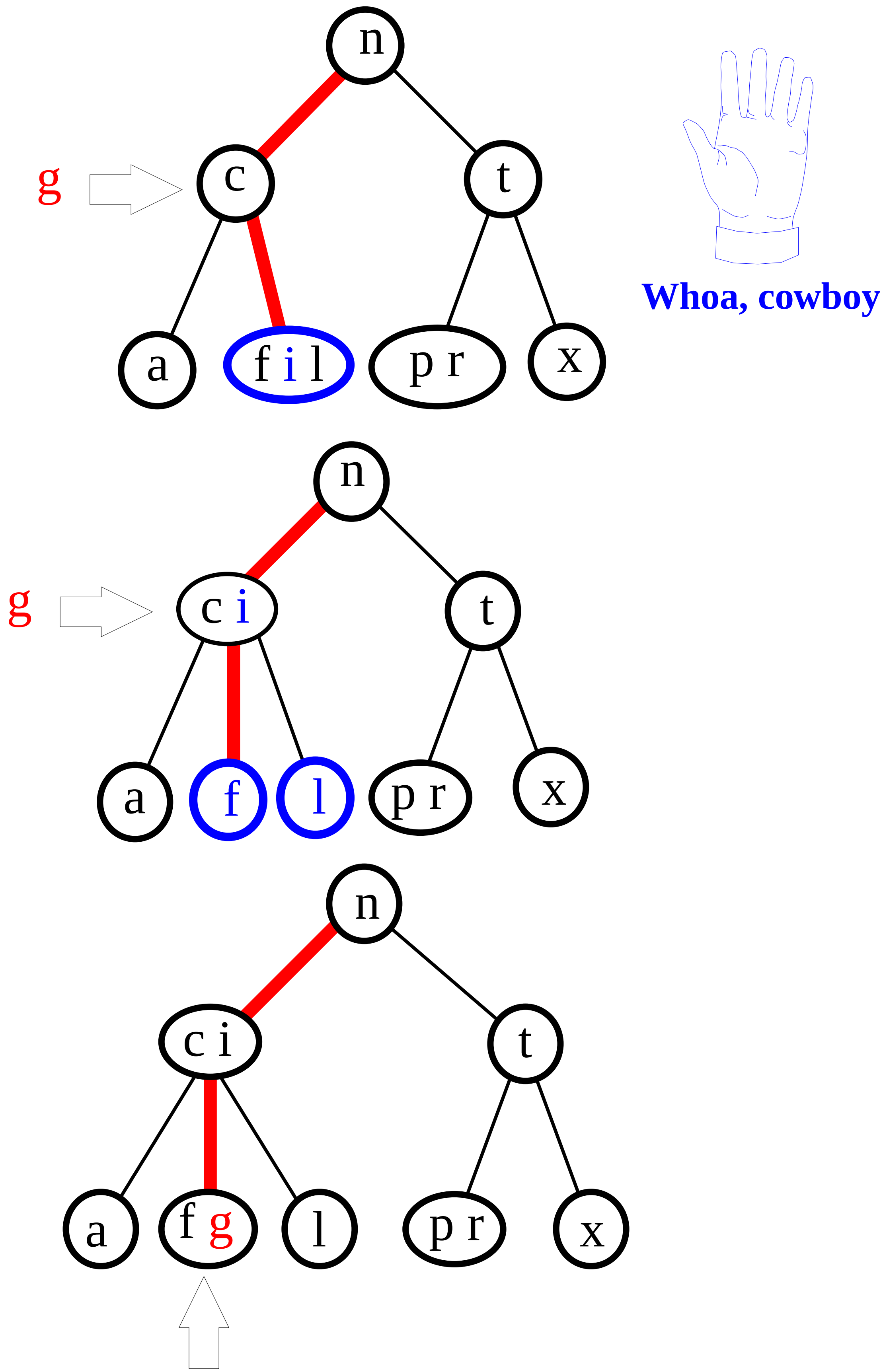


- Now we can perform the insertion using one of the previous two cases
- Since we follow this method from the root down to the leaf, it is called ***top down insertion***

An Example



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Time Complexity of Insertion in (2,4) Trees

Time complexity:

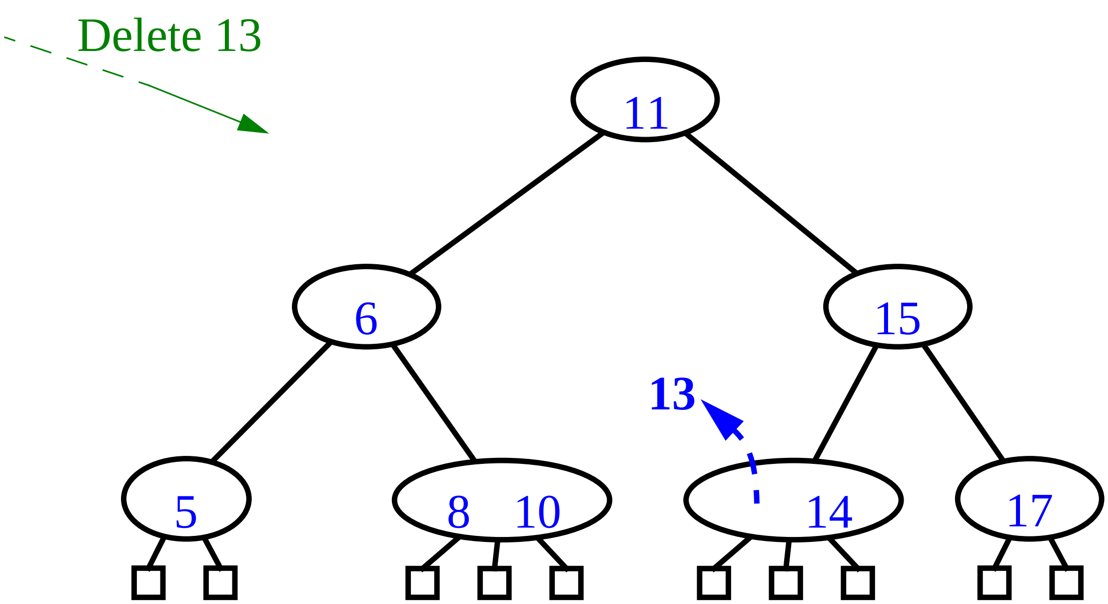
- A search visits O(log N) nodes
- An insertion requires O(log N) node splits
- Each node split takes constant time
- Hence, operations ***Search*** and ***Insert*** each take time O(log N)

Notes:

- Instead of doing splits top-down, we can perform them bottom-up starting at the insertion node, and only when needed. This is called ***bottom-up*** insertion.
- A deletion can be performed by ***fusing*** nodes (inverse of splitting), and takes O(log N) time. Let's take a look!

(2,4) Deletion

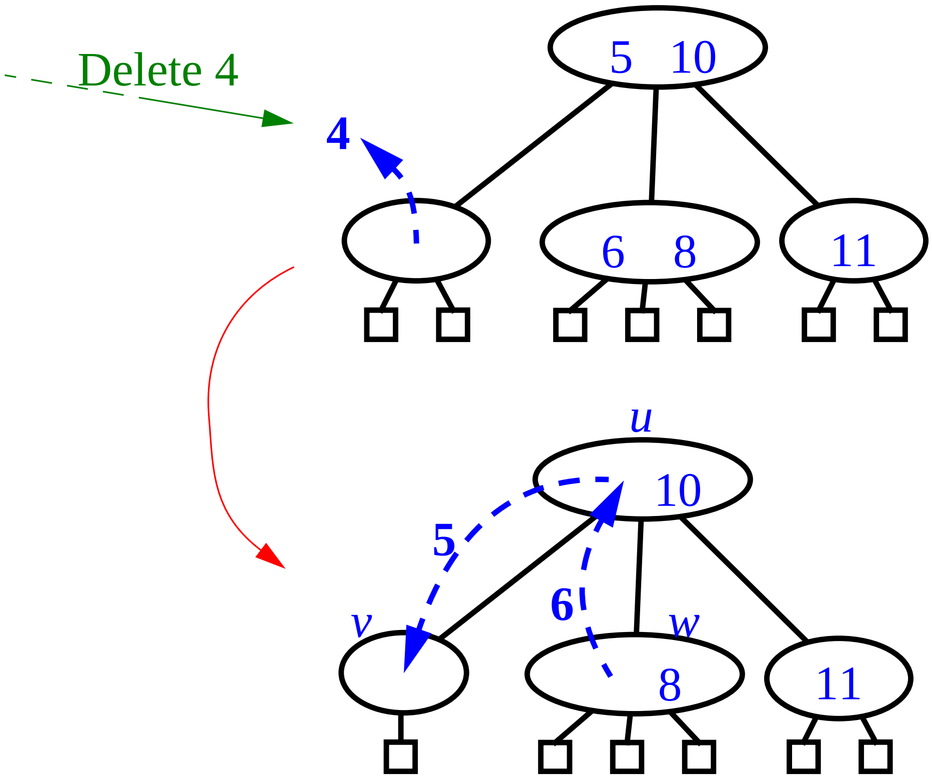
- A little trickier
- First of all, find the key
  - simple multi-way search
- If the item to delete has non-external children
  - reduce to the case where deletable item is at the bottom of the tree:
  - Find item which precedes it in in-order traversal
  - Swap them
- Remove the item



- Easy, right?
- ...but what about removing from 2-nodes?

(2,4) Deletion (cont.)

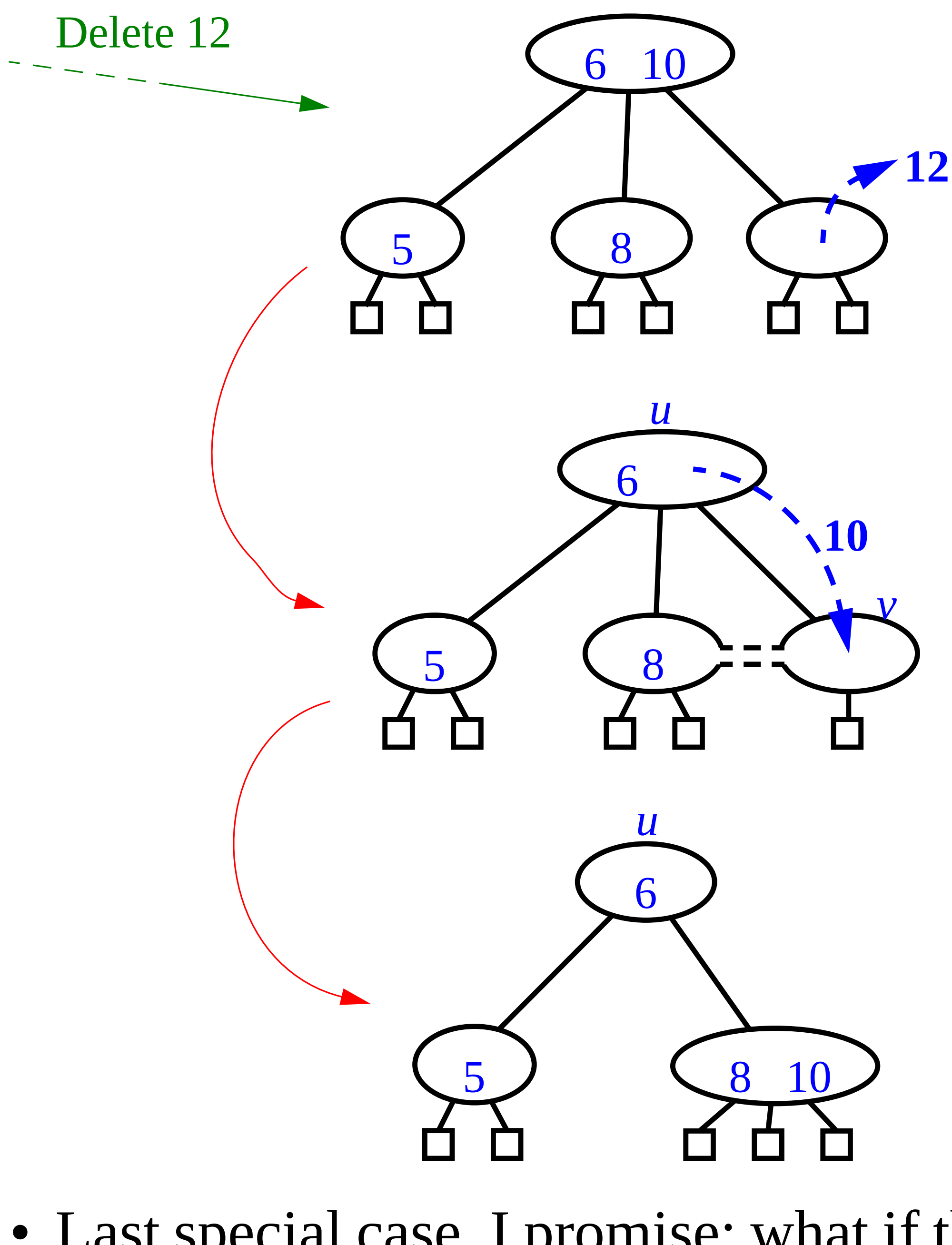
- Not enough items in the node
  - *underflow*
- Pull an item from the parent, replace it with an item from a sibling
  - called *transfer*



- Still not good enough! What happens if siblings are 2-nodes?
- Could we just pull one item from the parent?
  - too many children
- But maybe...

(2,4) Deletion (cont.)

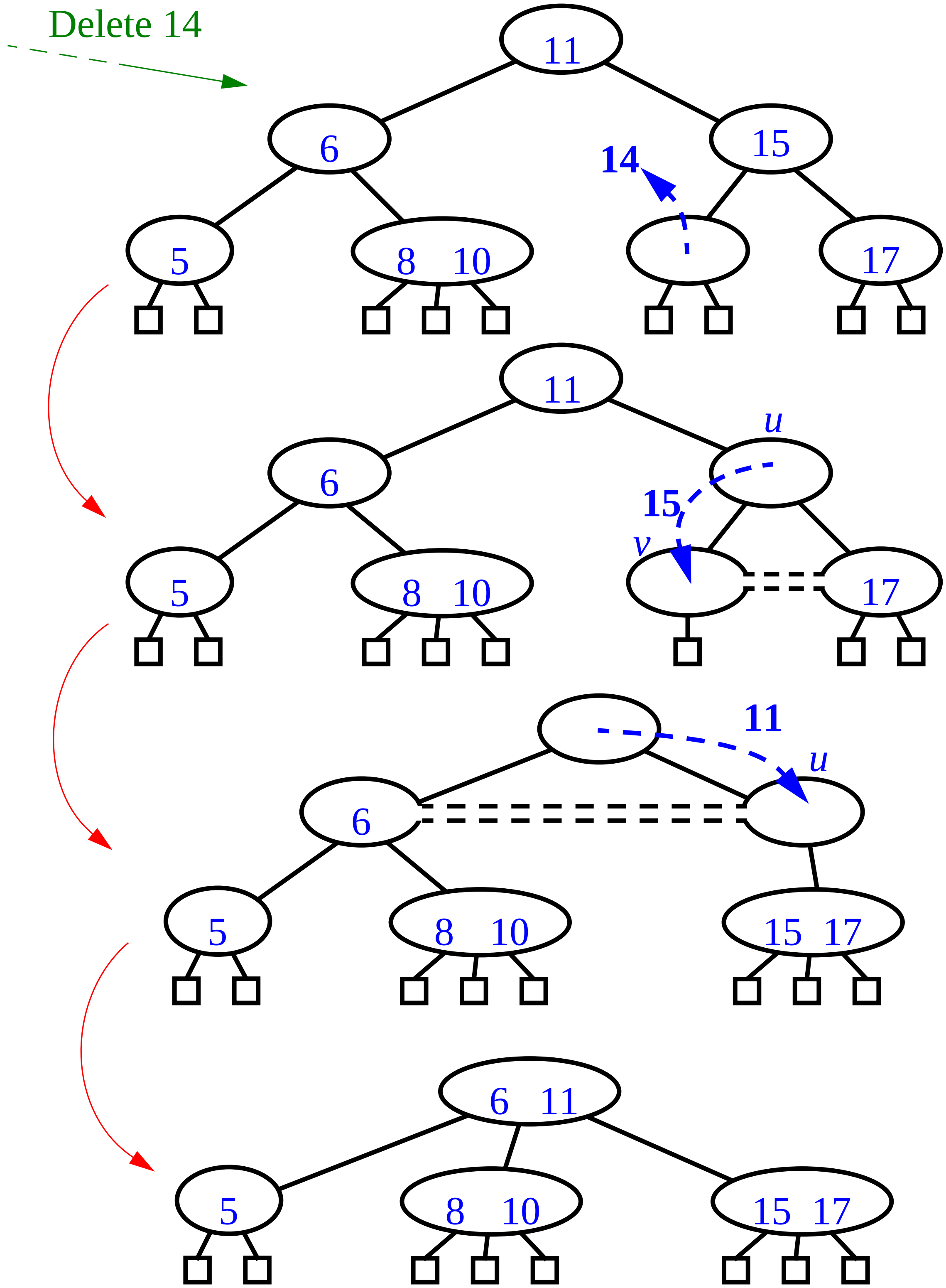
- We know that the node's sibling is just a 2-node
- So we *fuse* them into one
  - after stealing an item from the parent, of course



- Last special case, I promise: what if the parent was a 2-node?

(2,4) Deletion (cont.)

- Underflow can cascade up the tree, too.



(2,4) Conclusion

- The height of a (2,4) tree is $O(\log n)$.
- Split, transfer, and fusion each take $O(1)$.
- Search, insertion and deletion each take $O(\log n)$.
- Why are we doing this?
  - (2,4) trees are fun! Why else would we do it?
  - Well, there's another reason, too.
  - They're pretty fundamental to the idea of Red-Black trees as well.
  - And you're covering Red-Black trees on Monday.
  - Perhaps more importantly, your next project is a Red-Black tree.
- Have a nice weekend!