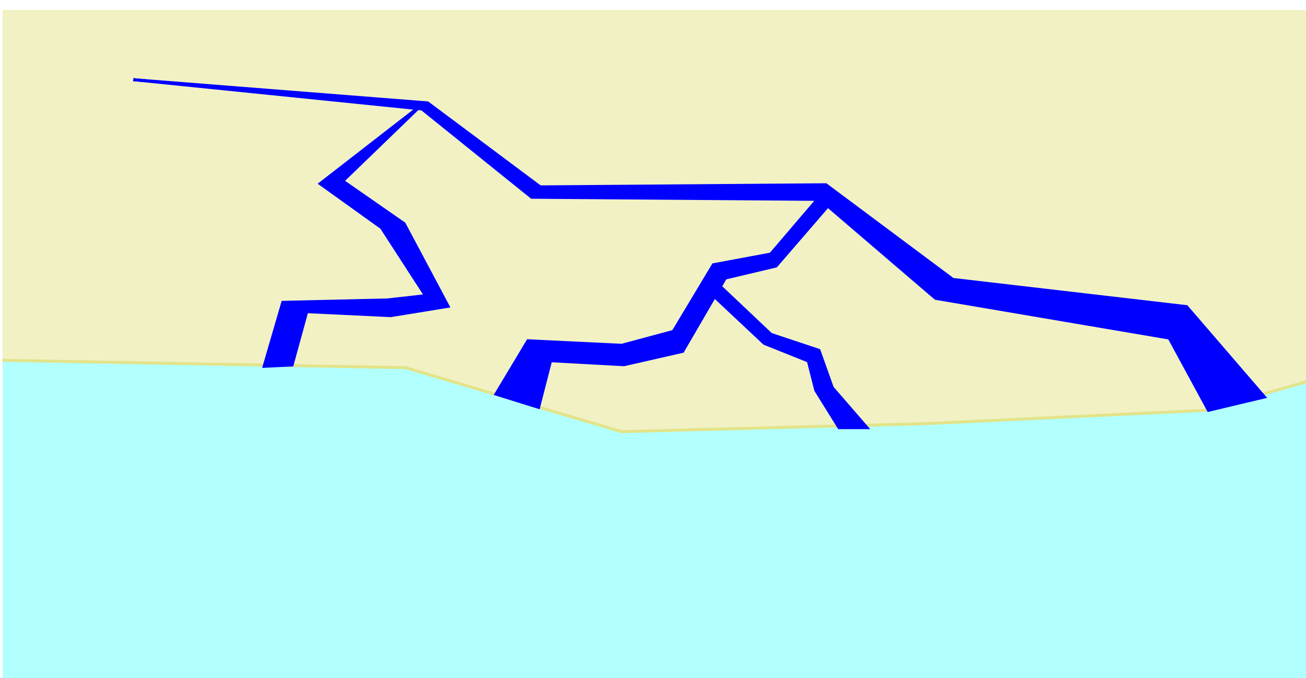
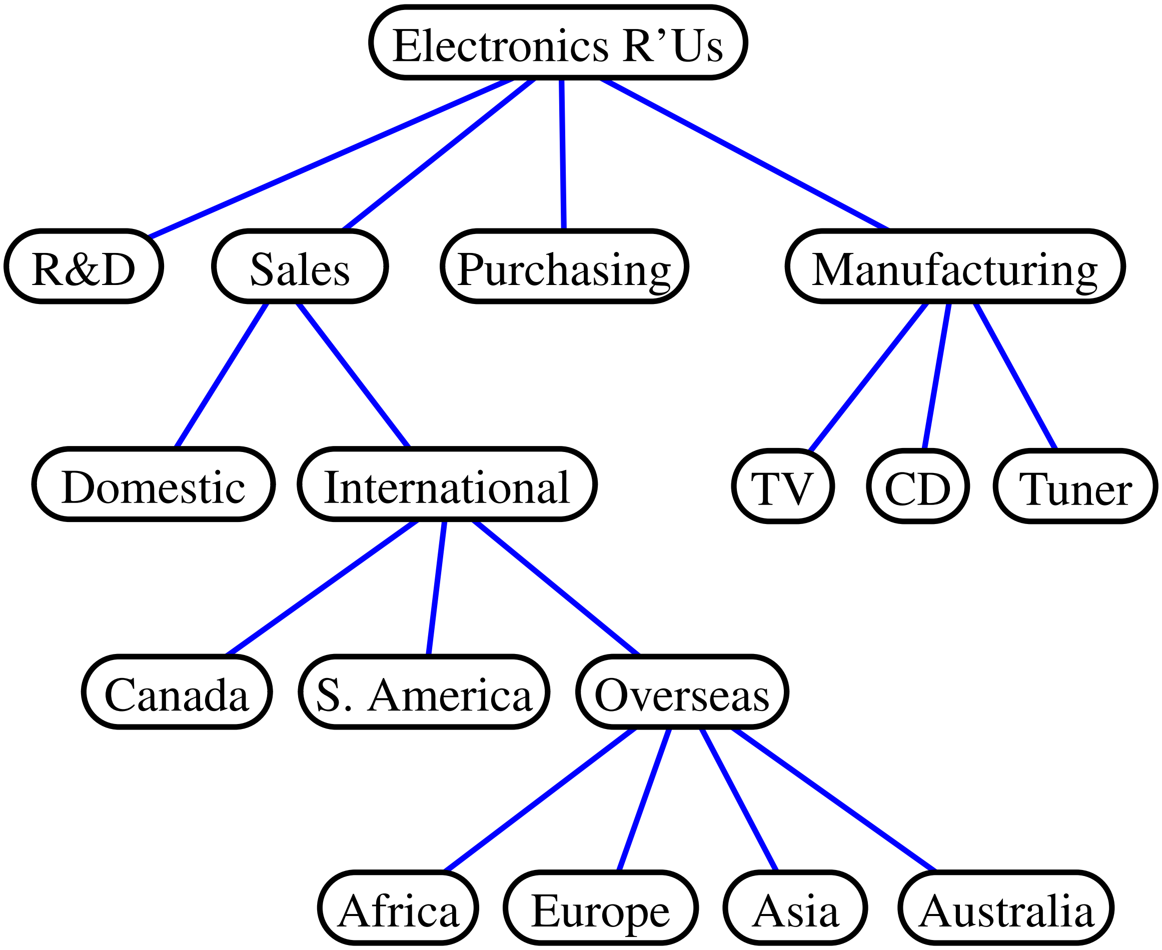
TREES

- trees
- binary trees
- traversals of trees
- template method pattern
- data structures for trees

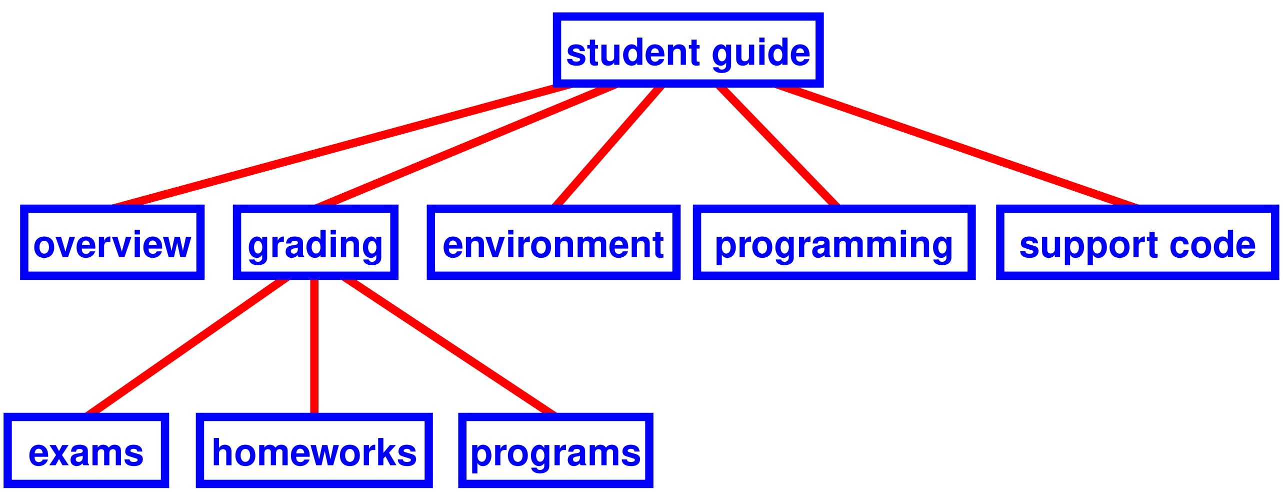


Trees

- a tree represents a hierarchy
  - organization structure of a corporation

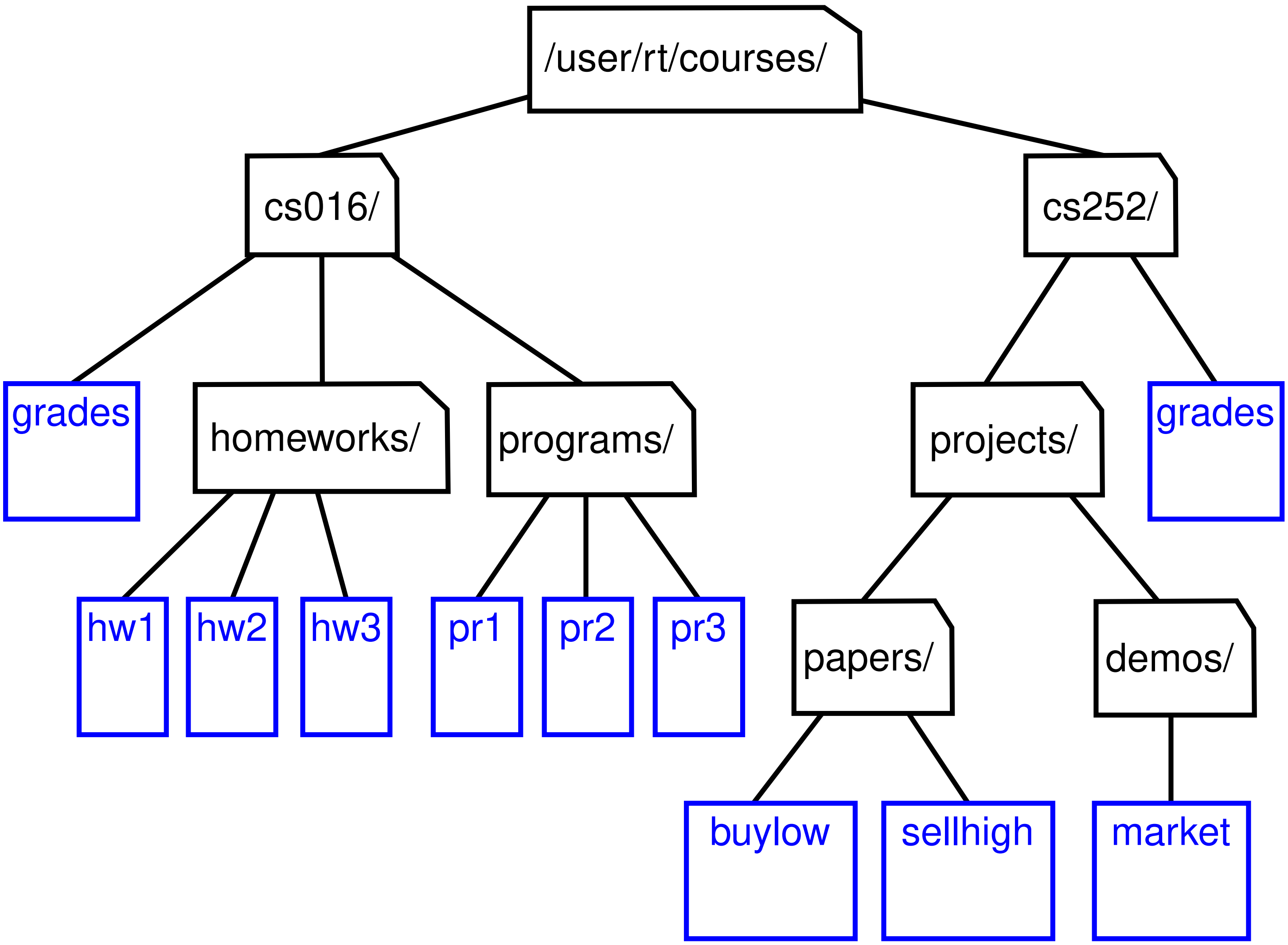


  - table of contents of a book



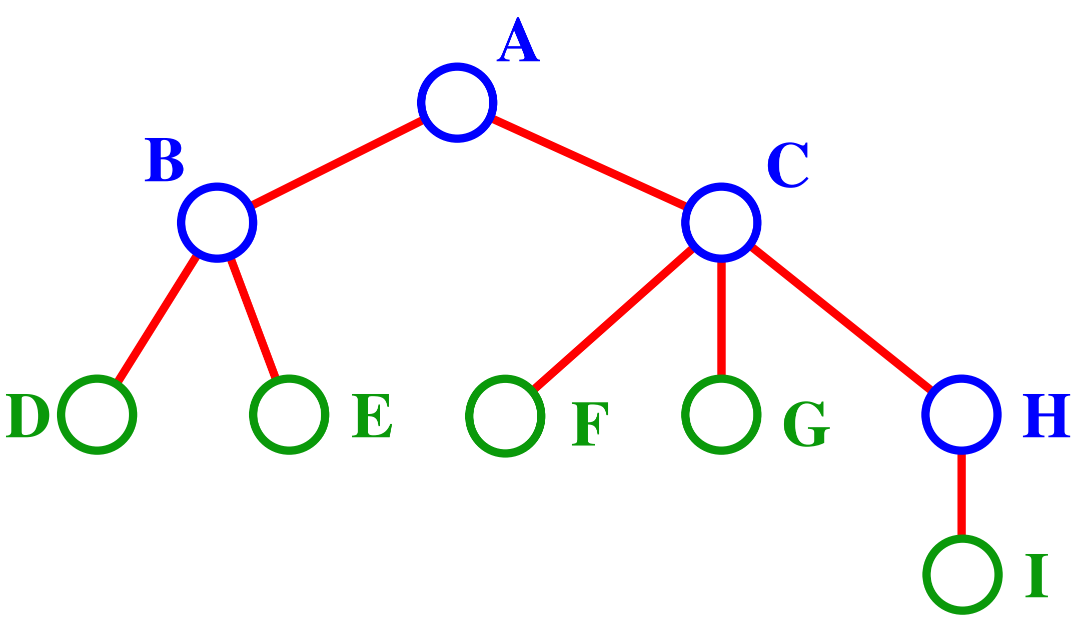
Another Example

- Unix or DOS/Windows file system



Terminology

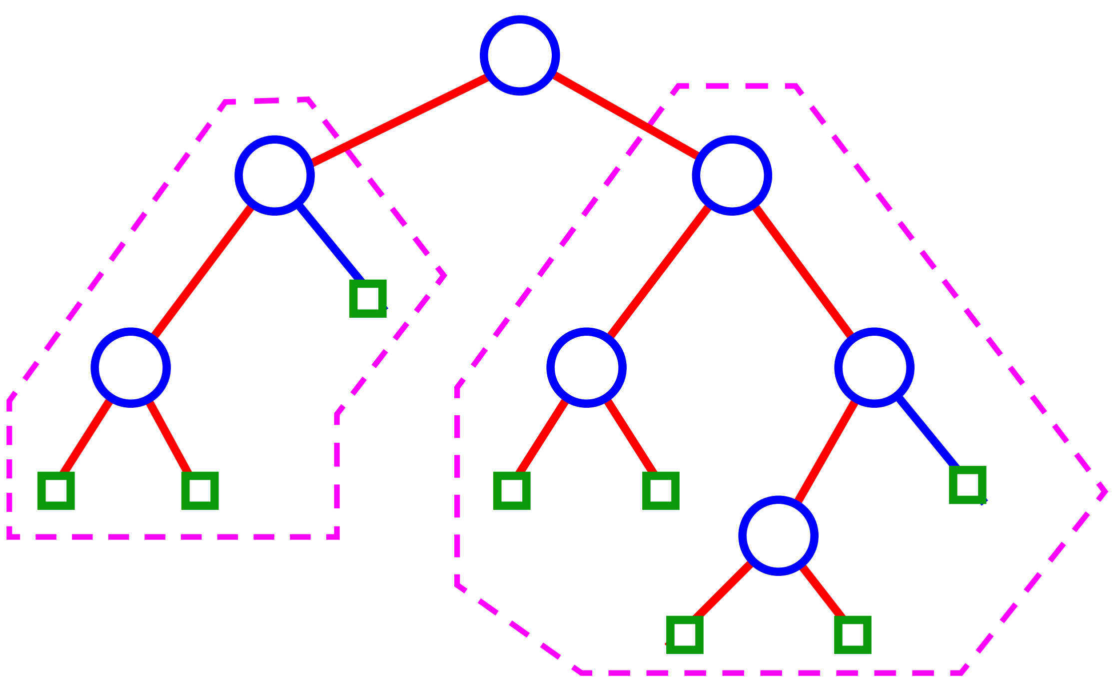
- ***A*** is the ***root*** node.
- ***B*** is the ***parent*** of D and E.
- ***C*** is the ***sibling*** of B
- ***D*** and ***E*** are the ***children*** of B.
- ***D, E, F, G***, I are ***external nodes*, or *leaves***.
- ***A, B, C, H*** are ***internal nodes*.**
- The ***depth*** (***level***) of ***E*** is ***2***
- The ***height*** of the tree is ***3***.
- The ***degree*** of node ***B*** is ***2***.



Property: (*# edges*) = (*#nodes*) − 1

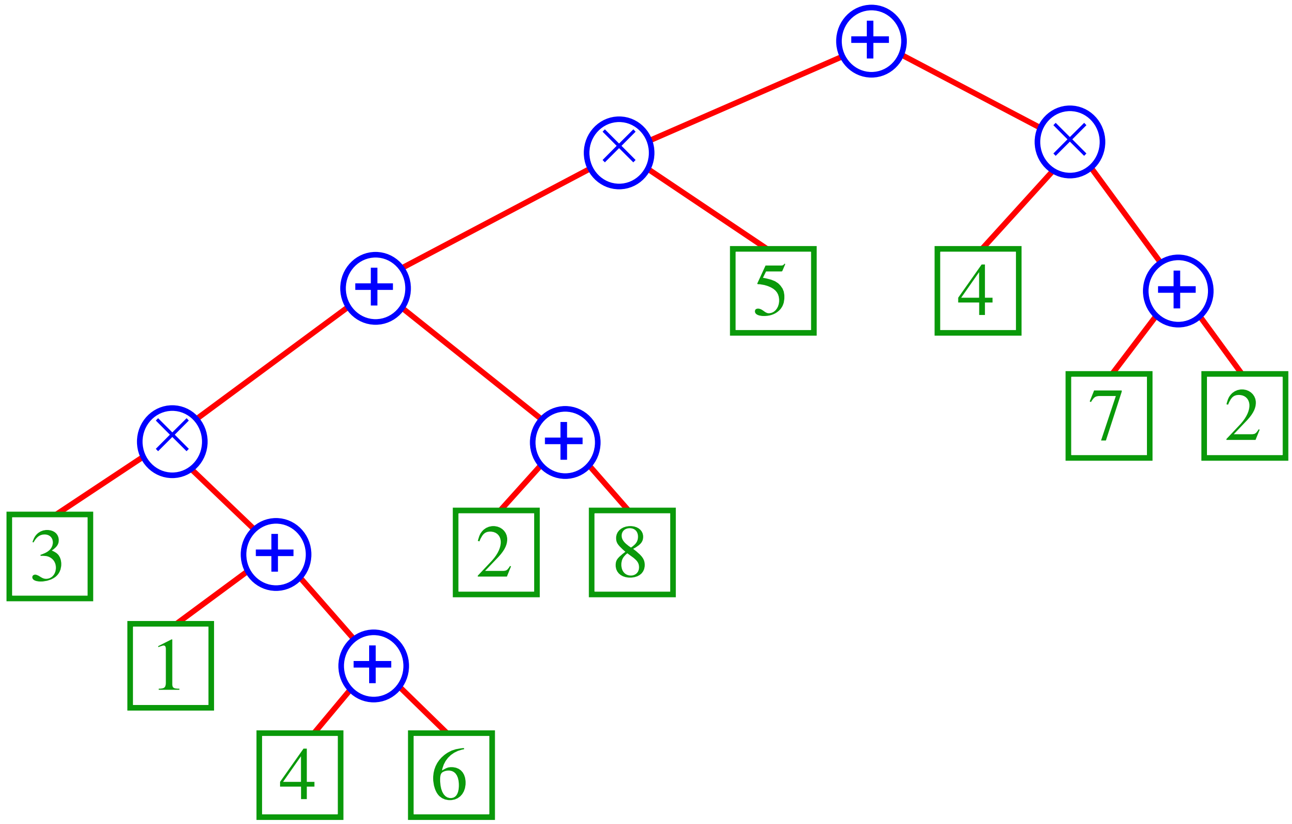
Binary Trees

- ***Ordered tree:*** the children of each node are ordered.
- ***Binary tree:*** ordered tree with all internal nodes of ***degree 2***.
- Recursive definition of binary tree:
- A ***binary tree*** is either
  - an external node (leaf), **or**
  - an internal node (the ***root***) and two binary trees (***left subtree*** and ***right subtree***)



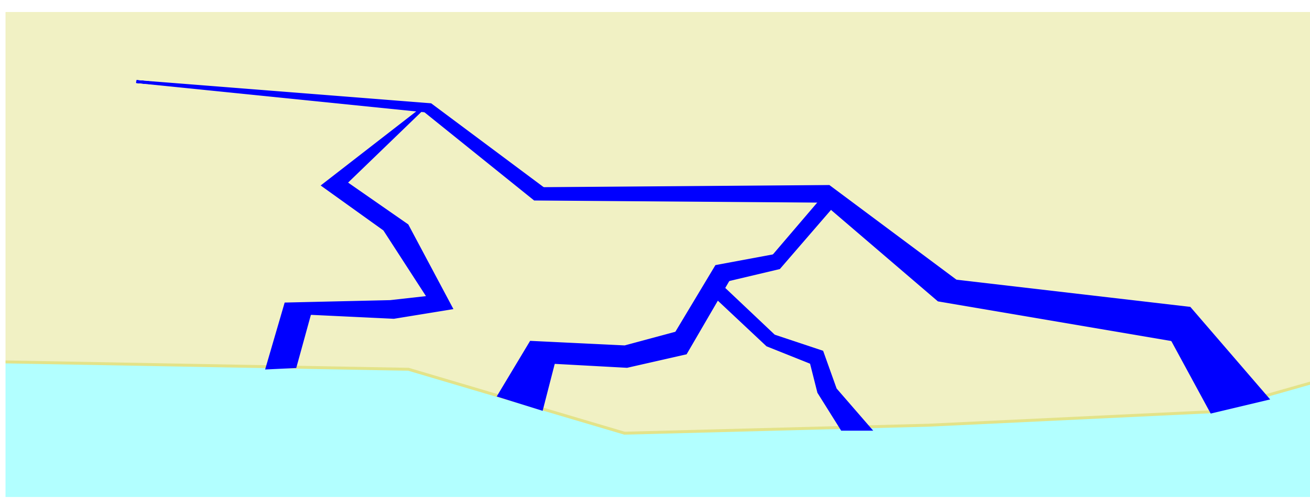
Examples of Binary Trees

- arithmetic expression



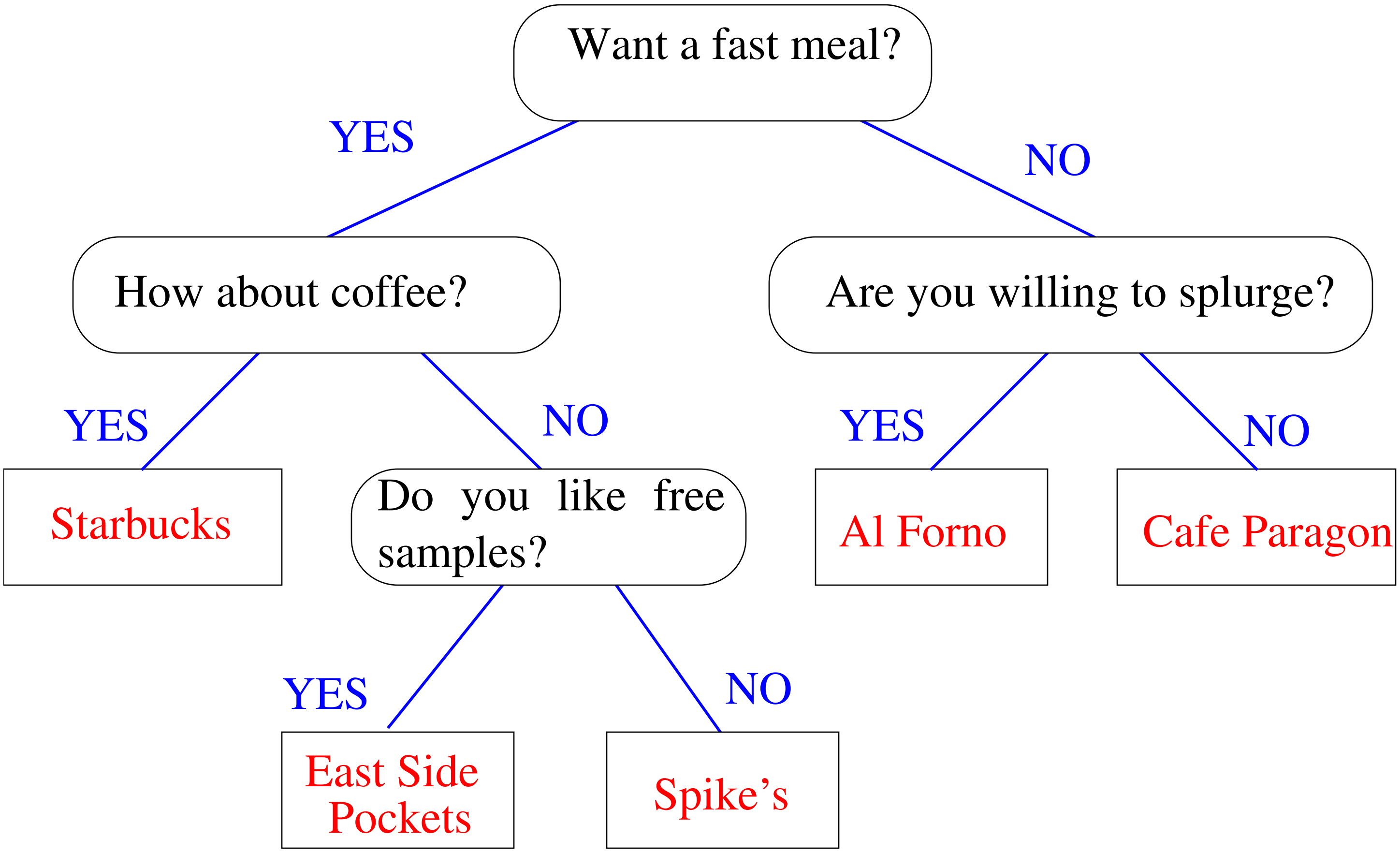
$((((3 \times (1 + (4 + 6))) + (2 + 8)) \times 5) + (4 \times (7 + 2)))$

- river



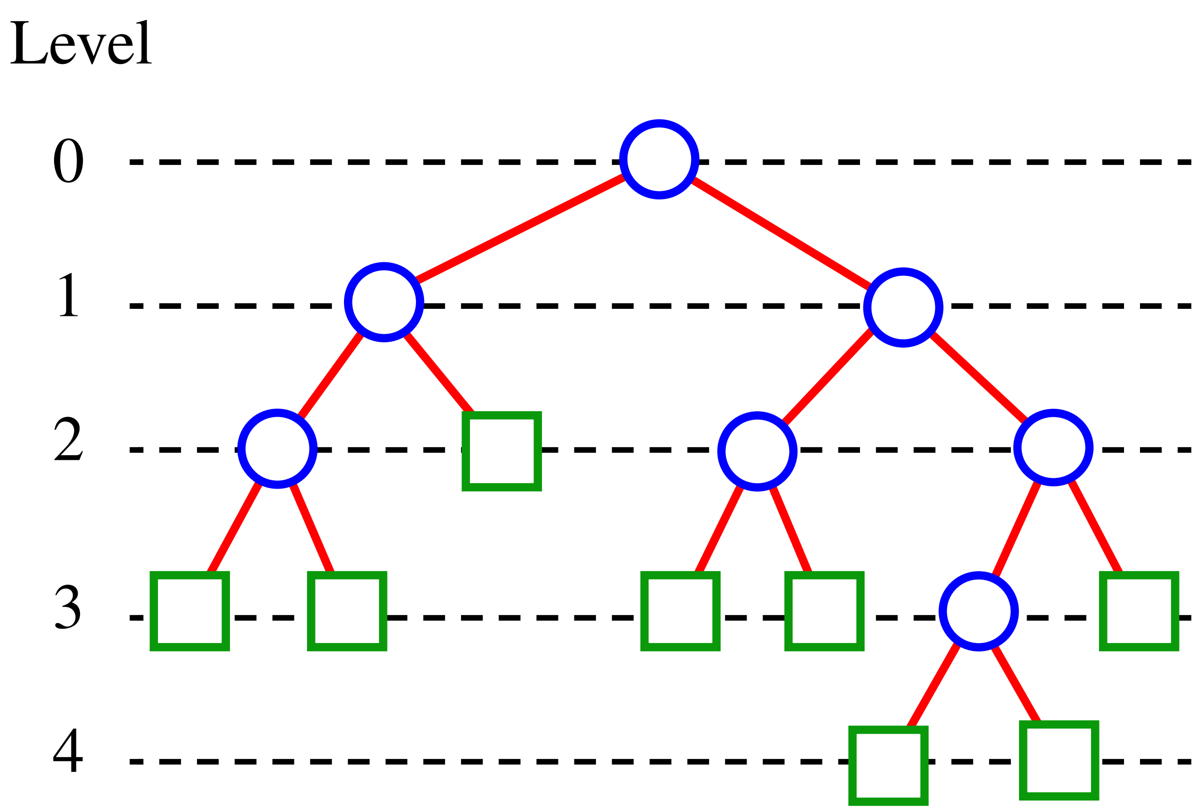
Examples of Binary Trees

- decision trees



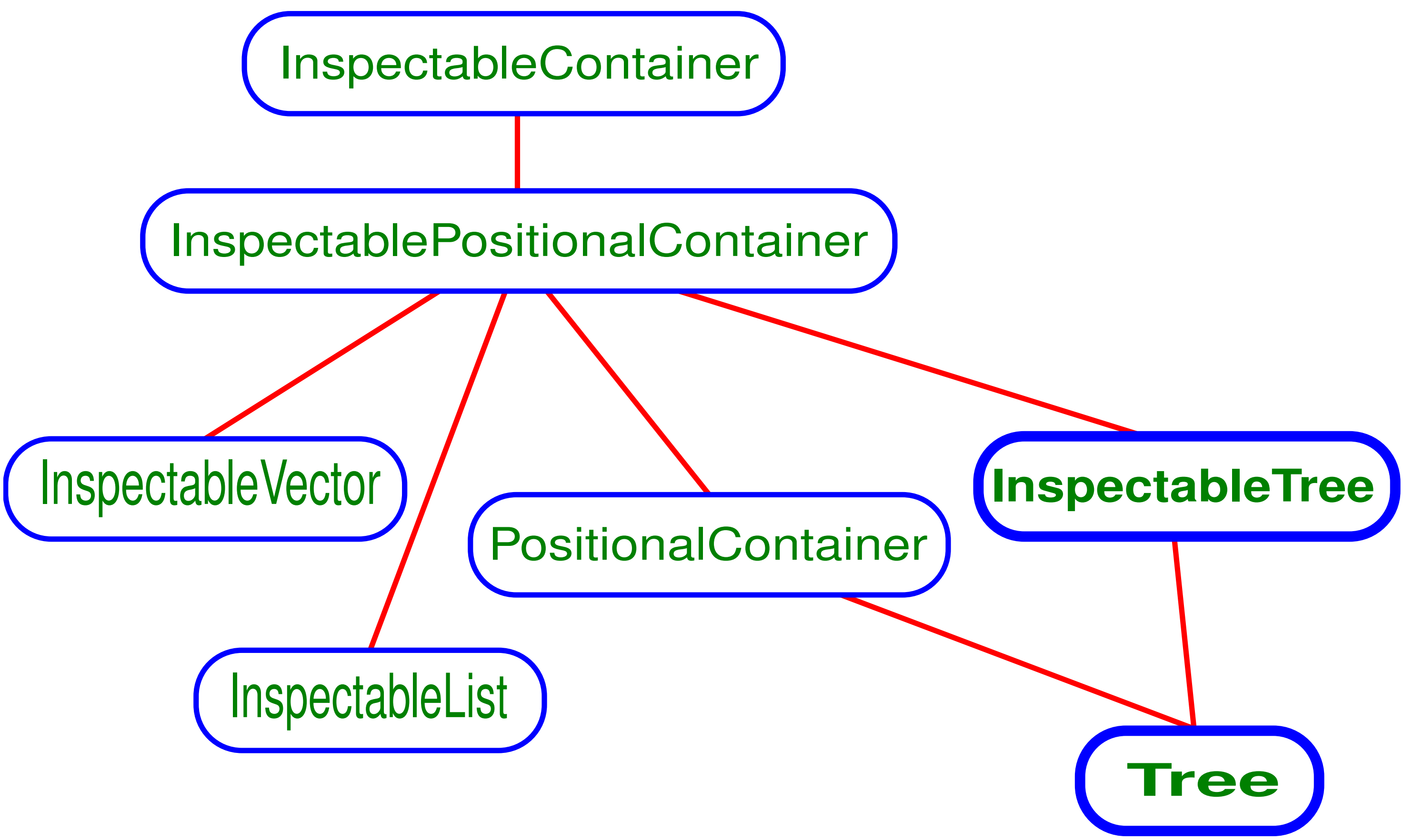
Properties of Binary Trees

- (# external nodes) = (# internal nodes) + 1
- (# nodes at level **i**) $\leq$ $2^{\mathbf{i}}$
- (# external nodes) $\leq$ $2^{(\text{height})}$
- (height) $\geq$ $\log_2$ (# external nodes)
- (height) $\geq$ $\log_2$ (# nodes) – 1
- (height) $\leq$ (# internal nodes) = ((# nodes) – 1)/2



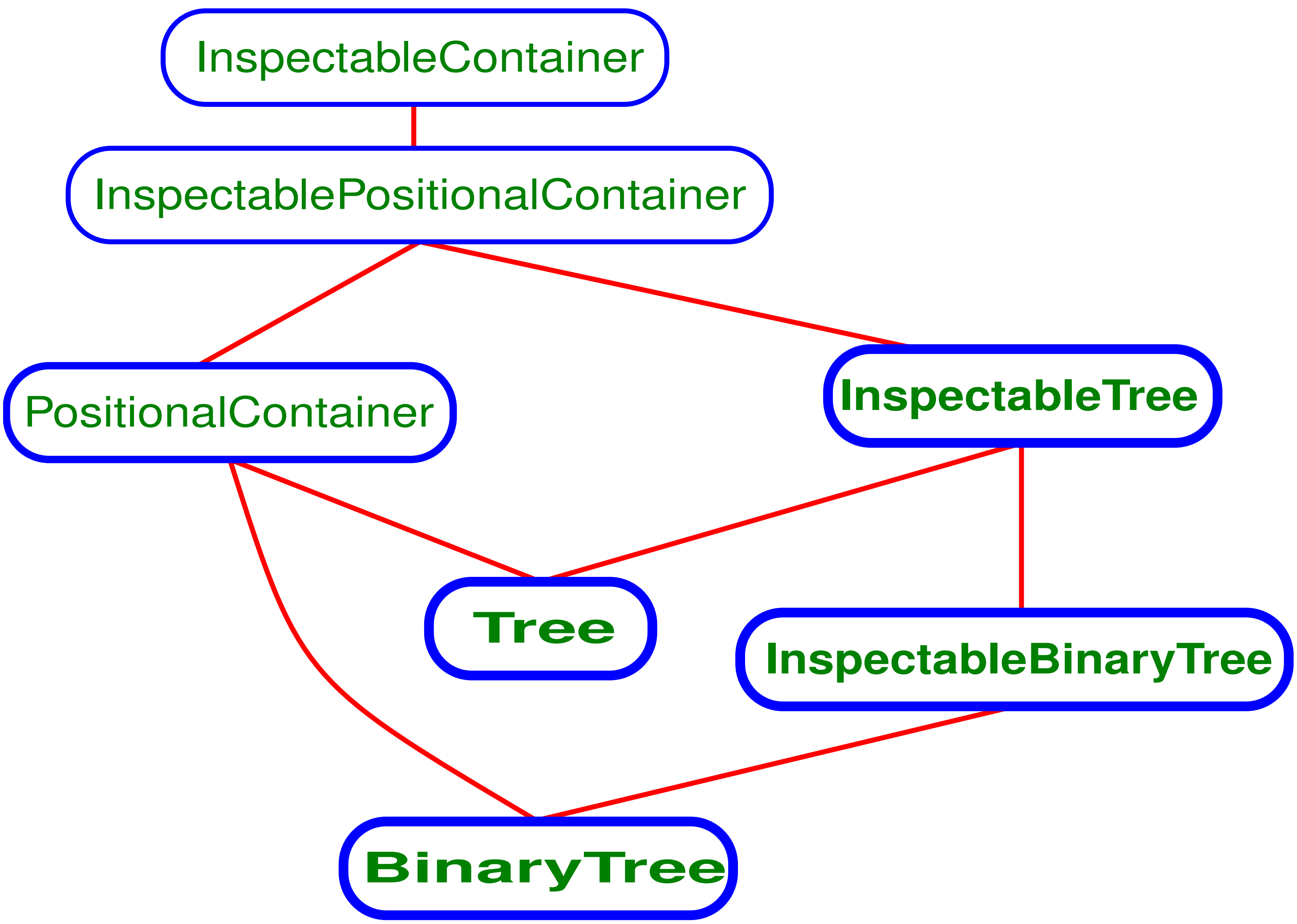
ADTs for Trees

- generic container methods
  - size(), isEmpty(), elements()
- positional container methods
  - positions(), swapElements(p,q), replaceElement(p,e)
- query methods
  - isRoot(p), isInternal(p), isExternal(p)
- accessor methods
  - root(), parent(p), children(p)
- update methods
  - application specific



ADTs for Binary Trees

- accessor methods
  - leftChild(p), rightChild(p), sibling(p)
- update methods
  - expandExternal(p), removeAboveExternal(p)
  - other application specific methods



Traversing Trees

- preorder traversal

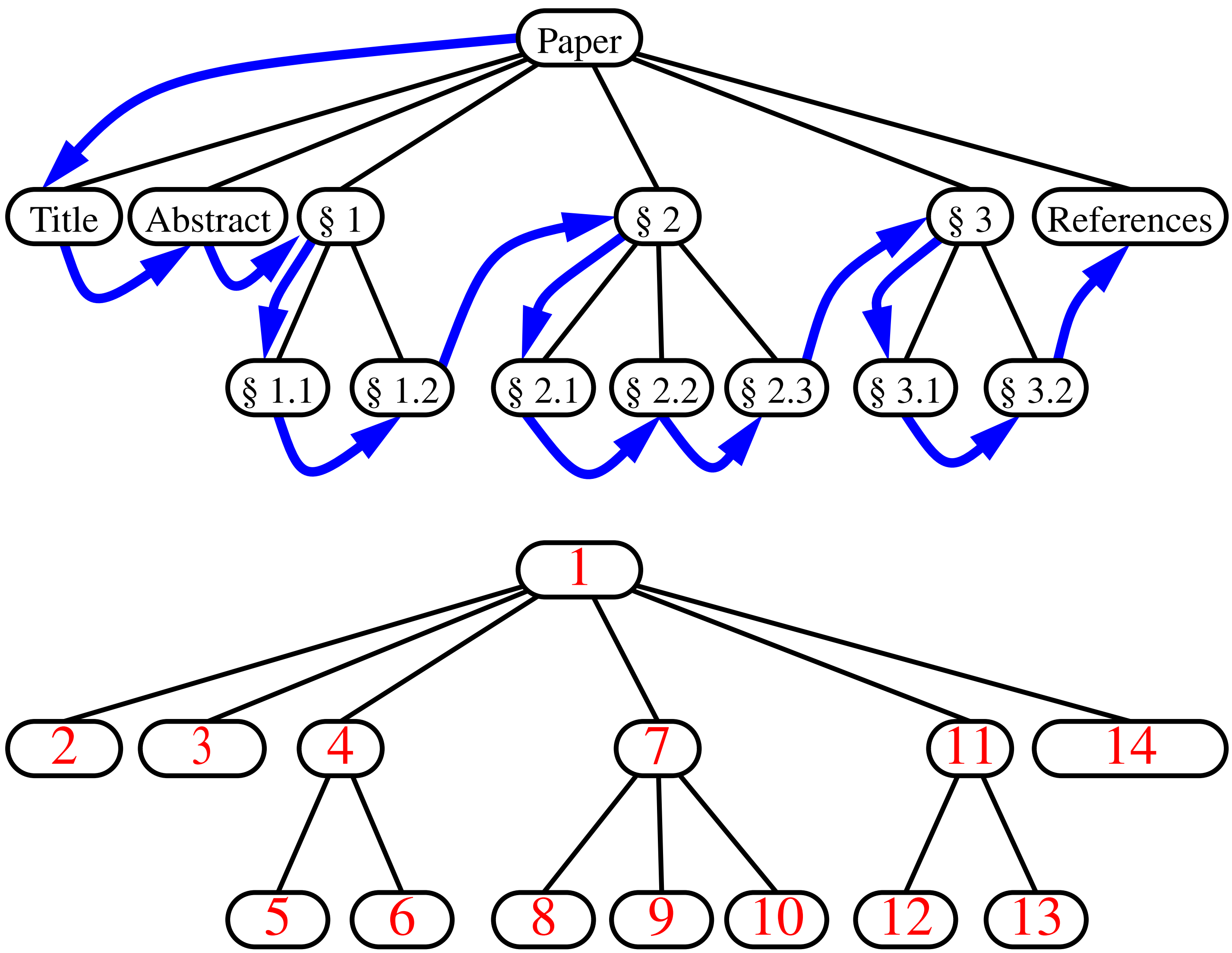
  Algorithm preOrder(v)

  "visit" node v

  for each child w of v **do**

  recursively perform preOrder(w)

- reading a document from beginning to end

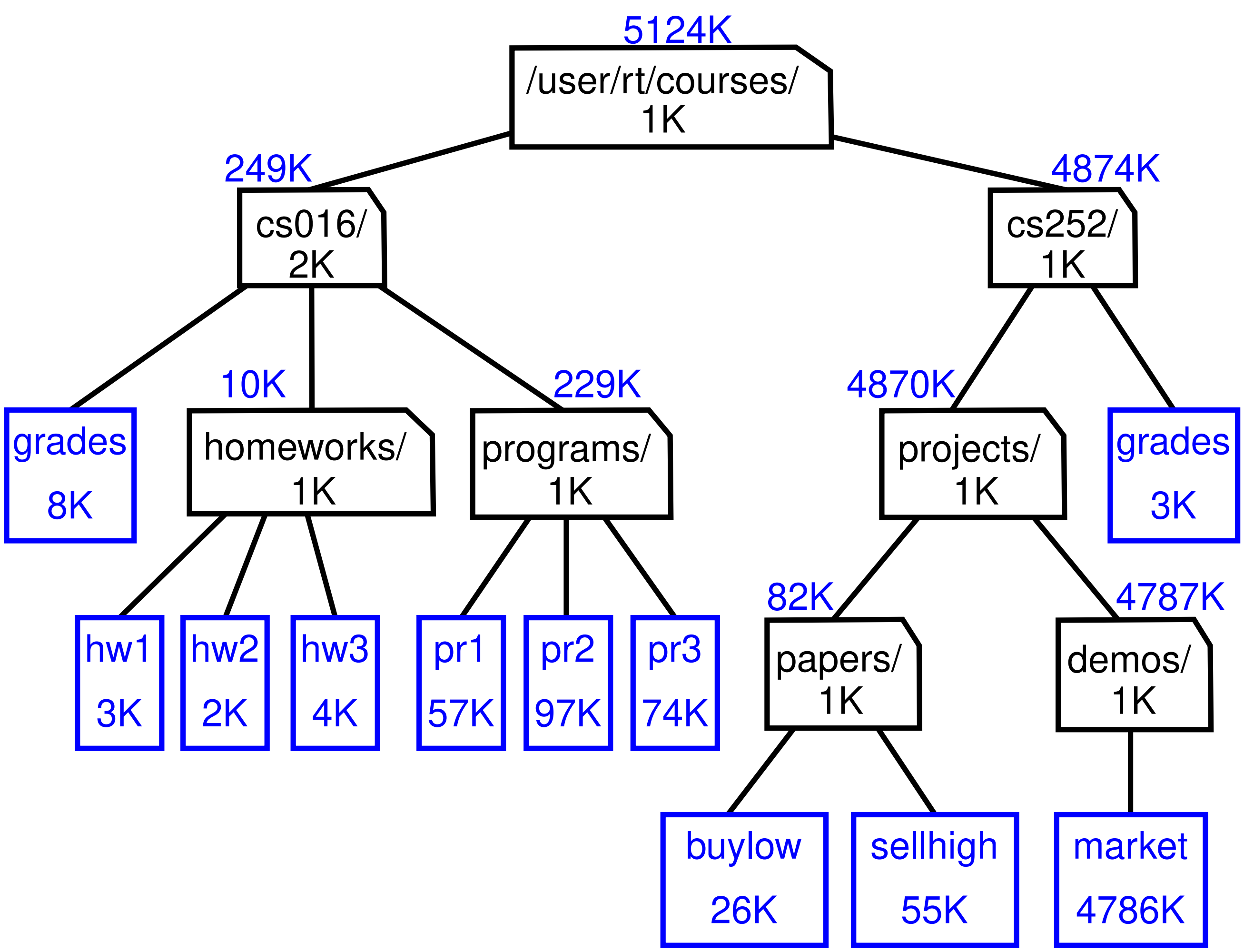


Traversing Trees

- postorder traversal

Algorithm postOrder(v)
  for each child w of v **do**
    recursively perform postOrder(w)
  "visit" node v

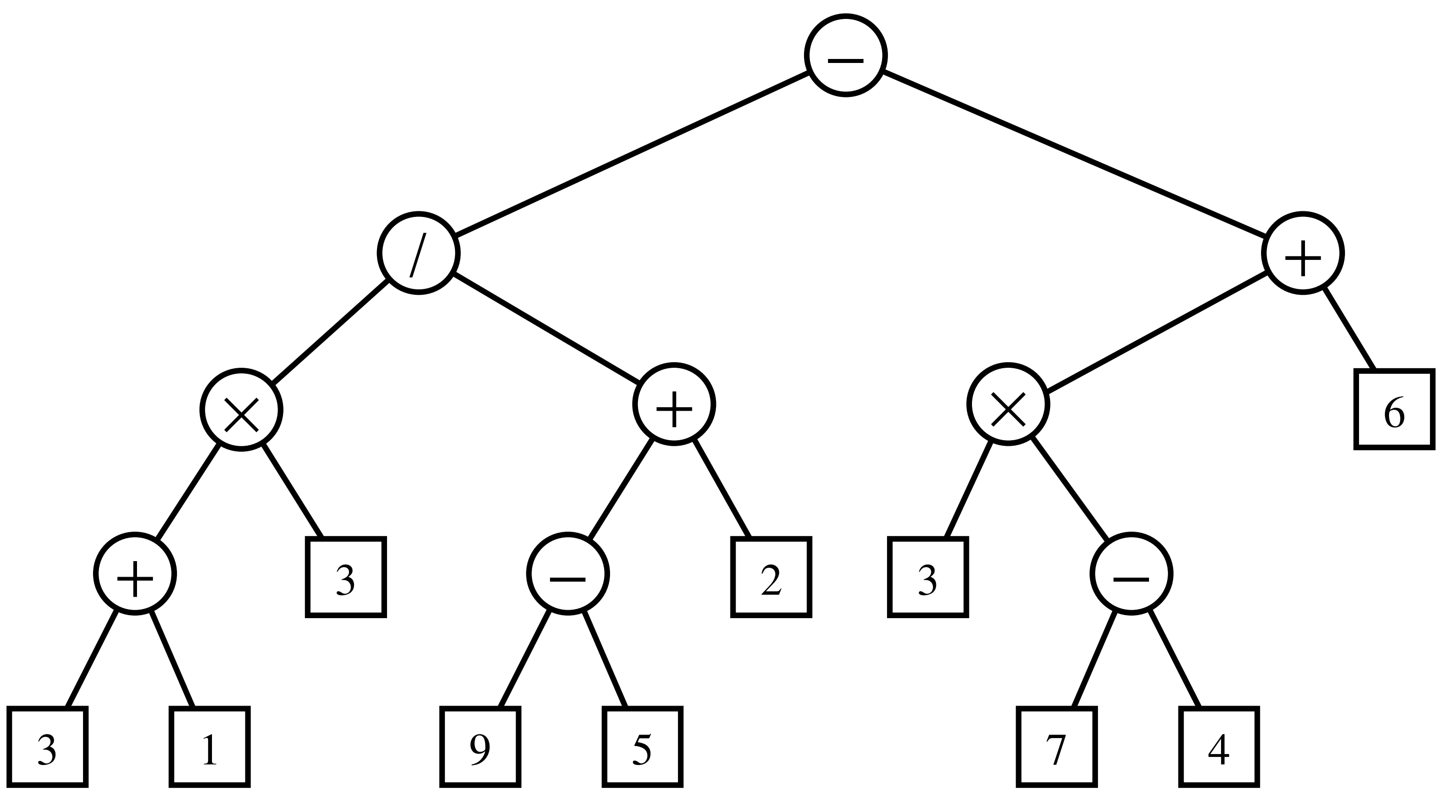
- du (disk usage) command in Unix



Evaluating Arithmetic Expressions

- specialization of a postorder traversal

Algorithm evaluateExpression(v)
  if v is an external node
    return the variable stored at v
  else
    let o be the operator stored at v
    x ← evaluateExpression(leftChild(v))
    y ← evaluateExpression(rightChild(v))
    return x o y



Traversing Trees

- inorder traversal of a binary tree

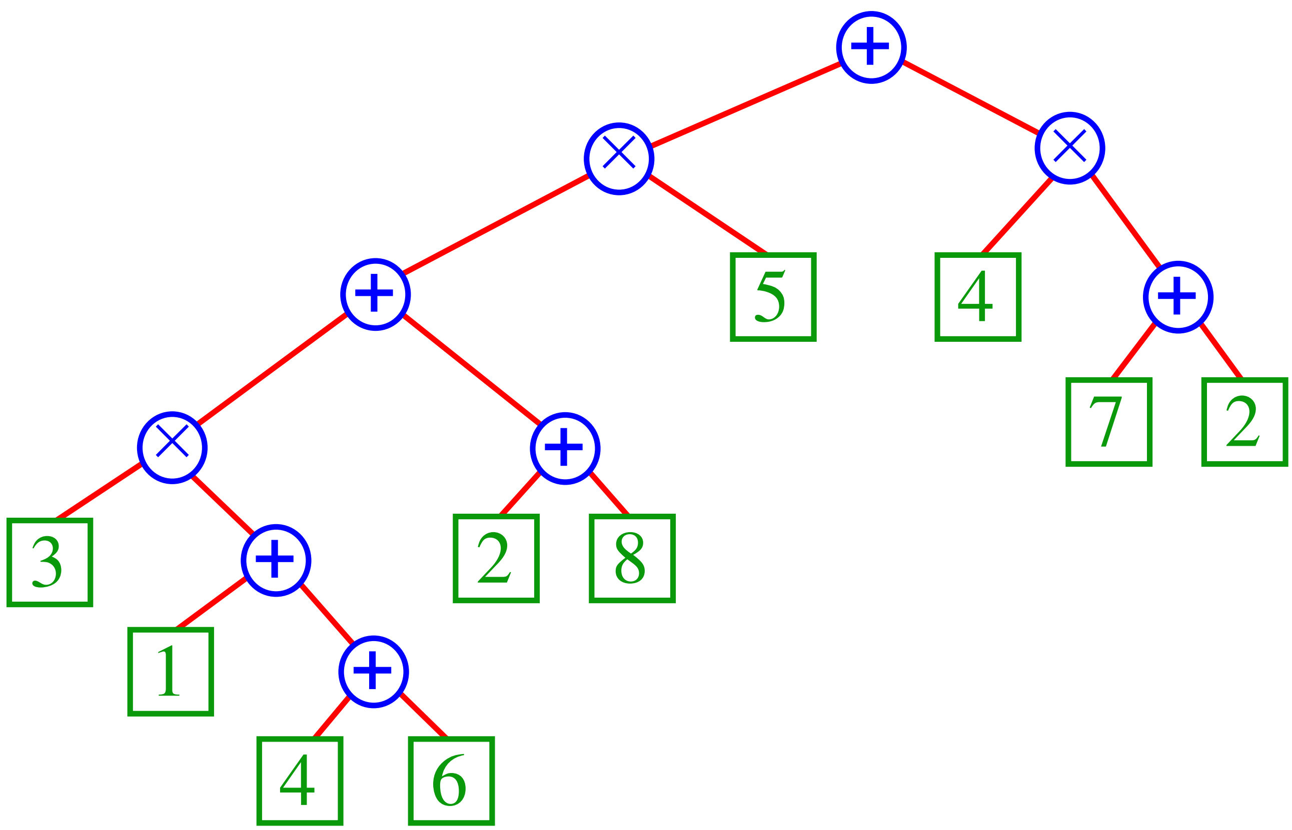
  Algorithm inOrder(v)

  recursively perform inOrder(leftChild(v))

  "visit" node v

  recursively perform inOrder(rightChild(v))

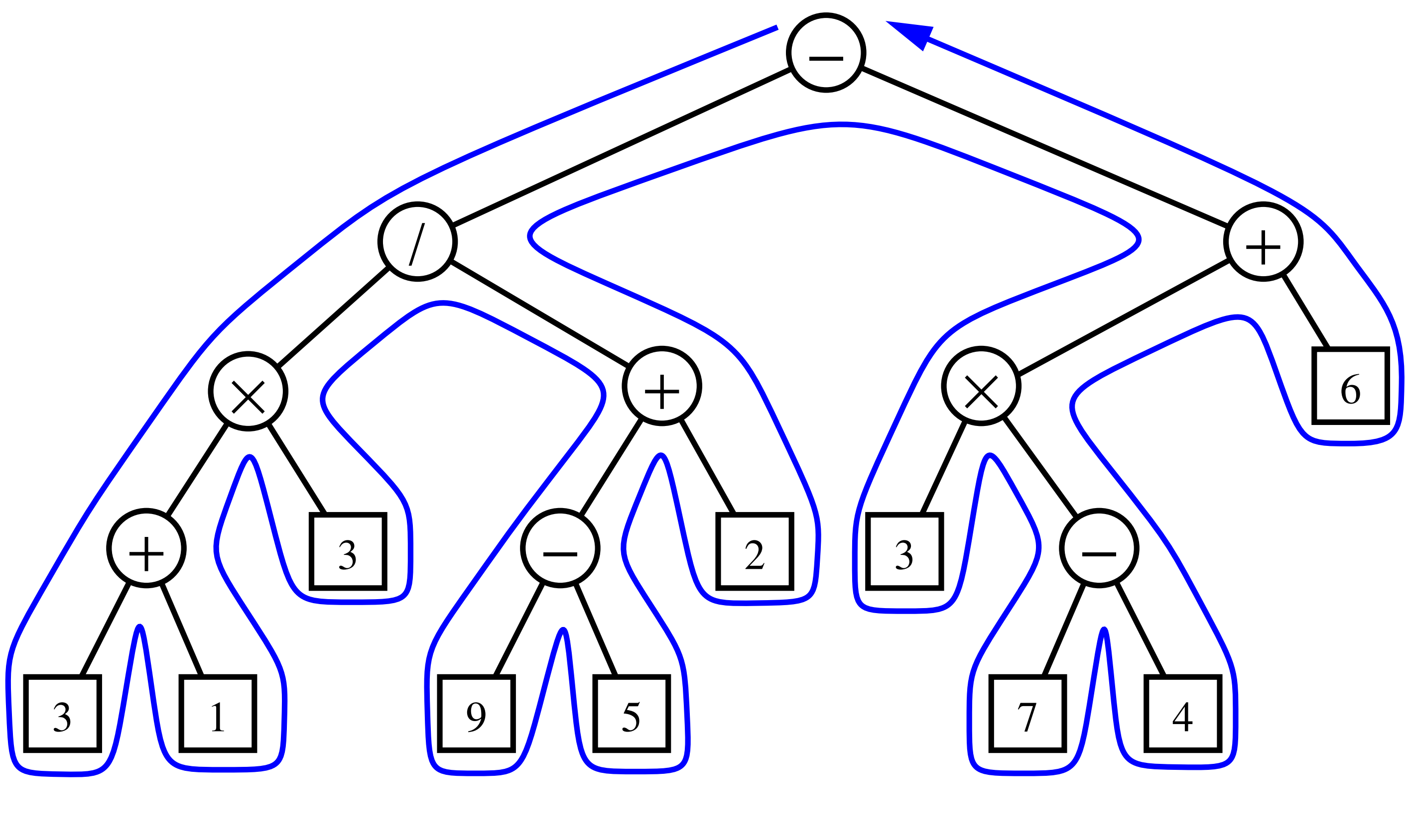
- printing an arithmetic expression
  - specialization of an inorder traversal
  - print "(" before traversing the left subtree
  - print ")" after traversing the right subtree



$((((3 \times (1 + (4 + 6))) + (2 + 8)) \times 5) + (4 \times (7 + 2)))$

Euler Tour Traversal

- generic traversal of a binary tree
- the preorder, inorder, and postorder traversals are special cases of the Euler tour traversal
- "walk around" the tree and visit each node three times:
  - on the left
  - from below
  - on the right



Template Method Pattern

- generic computation mechanism that can be specialized by redefining certain steps
- implemented by means of an abstract Java class with methods that can be redefined by it subclasses

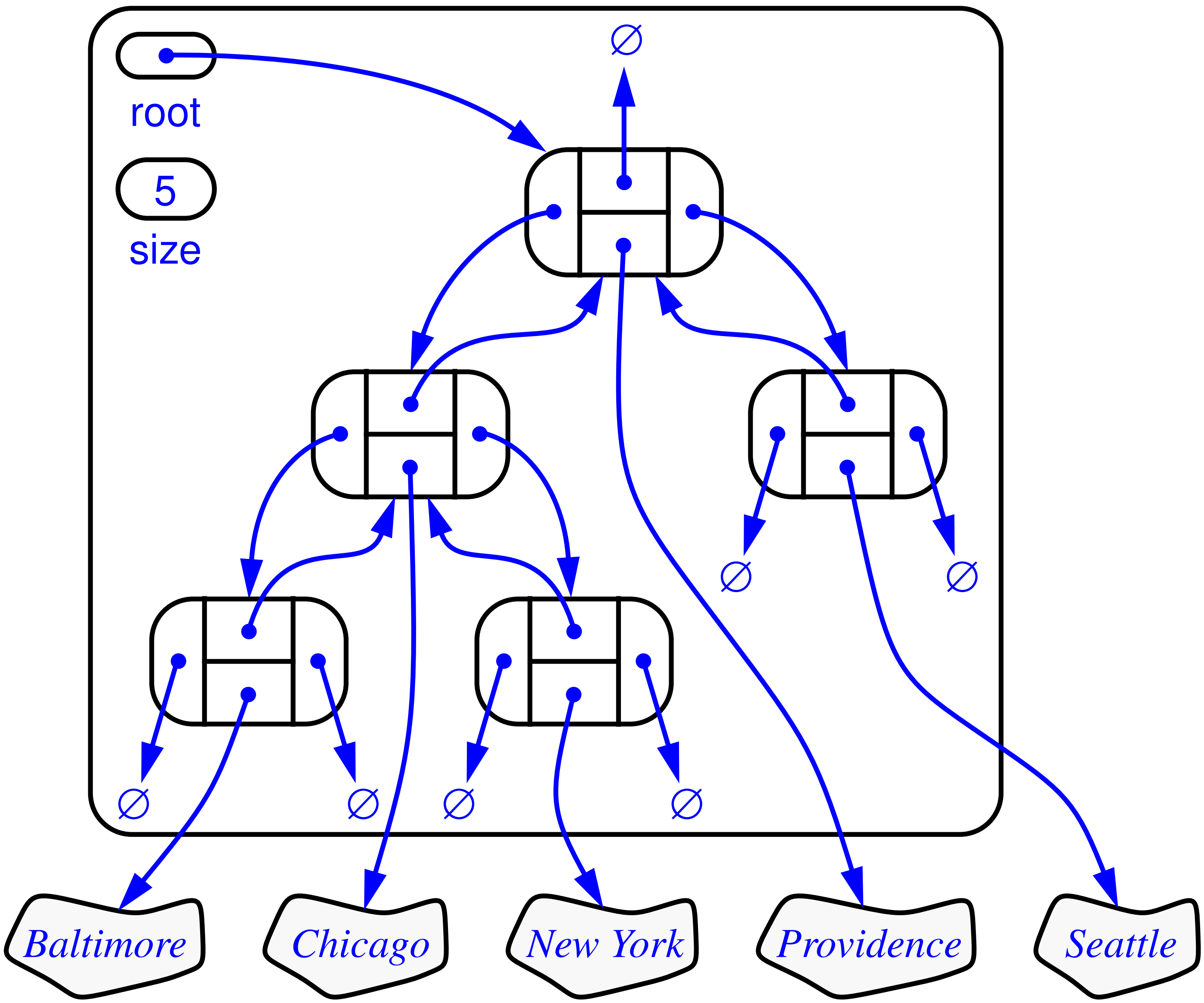
```
public abstract class BinaryTreeTraversal {

protected BinaryTree tree;

...

protected Object traverseNode(Position p) {
    TraversalResult r = initResult();
    if (tree.isExternal(p)) {
      external(p, r);
    } else {
      left(p, r);
      r.leftResult = traverseNode(tree.leftChild(p));
      below(p, r);
      r.rightResult = traverseNode(tree.rightChild(p));
      right(p, r);
    }
    return result(r);
  }
```

Specializing the Generic Binary Tree Traversal

- printing an arithmetic expression

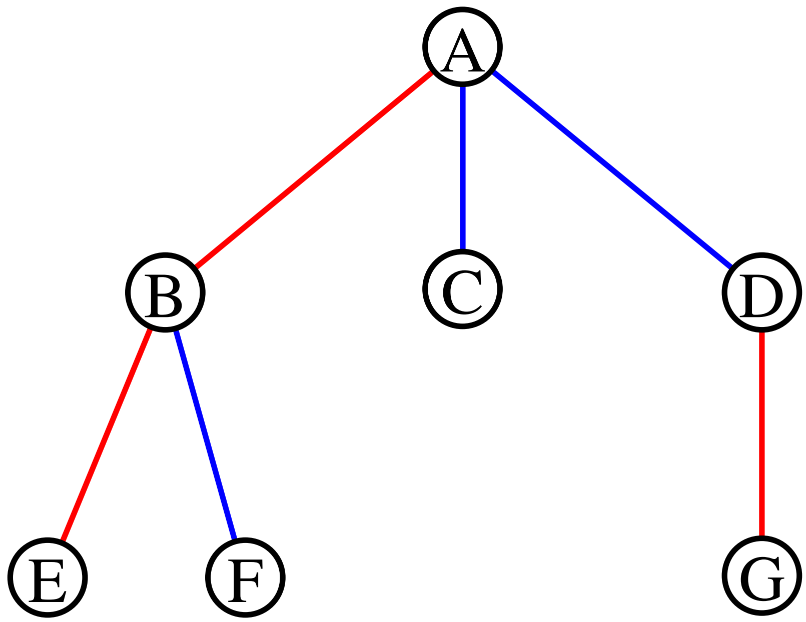
```
public class PrintExpressionTraversal
    extends BinaryTreeTraversal {

...

  protected void external(Position p, TraversalResult r) {
   System.out.print(p.element());
  }

  protected void left(Position p, TraversalResult r) {
   System.out.print("(");
  }

  protected void below(Position p, TraversalResult r) {
   System.out.print(p.element());
  }

  protected void right(Position p, TraversalResult r) {
   System.out.print(")");
  }

}
```

Linked Data Structure for Binary Trees



Representing General Trees

- tree T



- binary tree T' representing T