Trær

I. Eksempler, Definisjon

II. Binære Træer

III. Tre ADT

IV. Basis TreAlgoritmer (traversering)

V. Implementasjon av Binære Træer

Linket Struktur

Sequence (Array)

VI. Noen Anvendelser (???)

Ordbok Søking

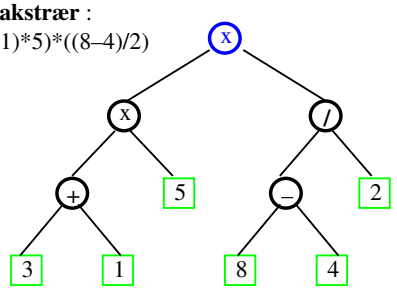
Streng Komprimering

Kap. 5 (kursorisk: 5.4.4; unntatt 5.5; + DFS/BFS)

Noen eksempler

Syntakstrær :

((3+1)*5)*((8–4)/2)

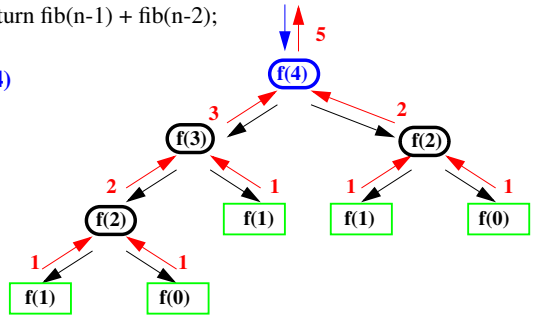


((3 + 1) x 5) x ((8 – 4) / 2)

Trær av rekursive kall :

```
int fib(int n) {
  if (n==0 || n==1) return 1;
  else
      return fib(n-1) + fib(n-2);
}
```

> **fib(4)**



Filhierarki :

> **cd**

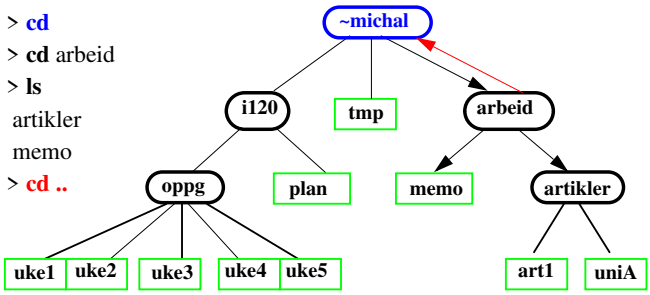
> **cd** arbeid

> **ls**

artikler

memo

> **cd ..**



en **Tre-struktur** T er enten :

en *Position*

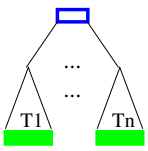
eller

en *Position r*

og

n disjunkte trær

(r ikke i Ti)



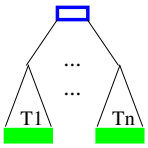
Definisjon og Terminologi

1. **r** er **roten** (posisjon) i **T**
2. **b** er **forelder** (parent) til **e**, **f**, **g** : umiddelbar forgjenger
   og **forgjenger** til **e**, **f**, **g**, samt **j**, **k**
3. **e**, **f**, **g** er **barna** til **b** : umiddelbare etterfølgere (og er **søsken**)
4. **d**, **f**, **g**, **h**, **i**, **j**, **k** **eksterne** posisjoner (**blader**) : de uten etterfølgere
5. **r**, **a**, **b**, **c**, **e** er **interne** posisjoner : ikke eksterne
6. **dybden** av **e** = 2 : lengden av stien fra roten (**nivå**)
   depth(p) = if isRoot(p) 0
   else 1+ depth(parent(p))
7. **høyden** av **b** = 2 : avstand til fjerneste bald under b
   height(p) = if isExternal(p) 0
   else 1+ max{ height(x) : x er bland children(p) }
   høyden av **T** = høyden av r(oten til T)
8. **graden** av **b** = 3 : antall barn
9. **T1**, **T2** er **subtrær** av **T**

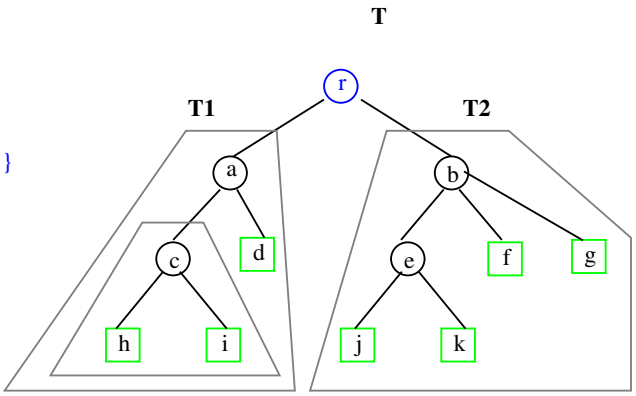
Noen egenskaper

10. (# kanter) = (# posisjoner) – 1
11. for hver posisjon finnes det en *entydig sti* til roten

et **Tre** T er enten :
en *Position* r
eller
en *Position* r og
n disjunkte subtrær
(r ikke i Ti)



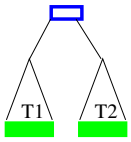
blad=min < subtre < tre



Binære Trær

graden til enhver *Pos.* er **2** el. **0**

et **Binær-Tre** er enten :
en *Position* r
eller
en *Position* r
og
2 disjunkte subtrær
(r ikke i Ti)



av og til : **høyst 2** disjunkte subtrær

Noen egenskaper (h: høyde; n: #noder)

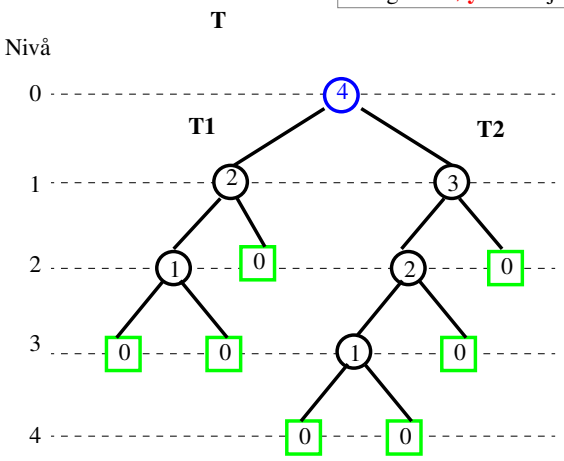
A. (# eksterne noder) = (# interne noder) +1

B. (# noder på nivå i) $\leq 2^i$

C. $h+1 \leq$ (# eksterne noder) $\leq 2^h$
   $\log_2$ (# eksterne noder) $\leq h$

D. $h \leq$ (# interne noder) $\leq 2^h-1$
   $\log_2$ (# interne noder) $\leq h$

E. $2h+1 \leq n \leq 2^{(h+1)}-1$
   $\log_2(n+1) - 1 \leq h \leq (n-1)/2$



package jdsl.core.api;

```
public interface Container
{ Enumeration elements();
  boolean isEmpty();
  int size();
  Container newConta
}
```

```
public interface PositionalContainer  extends Container
{ Enumeration positions();
  void swap(Position p, Position q);
  Object replace(Position p, Object e);
}
```

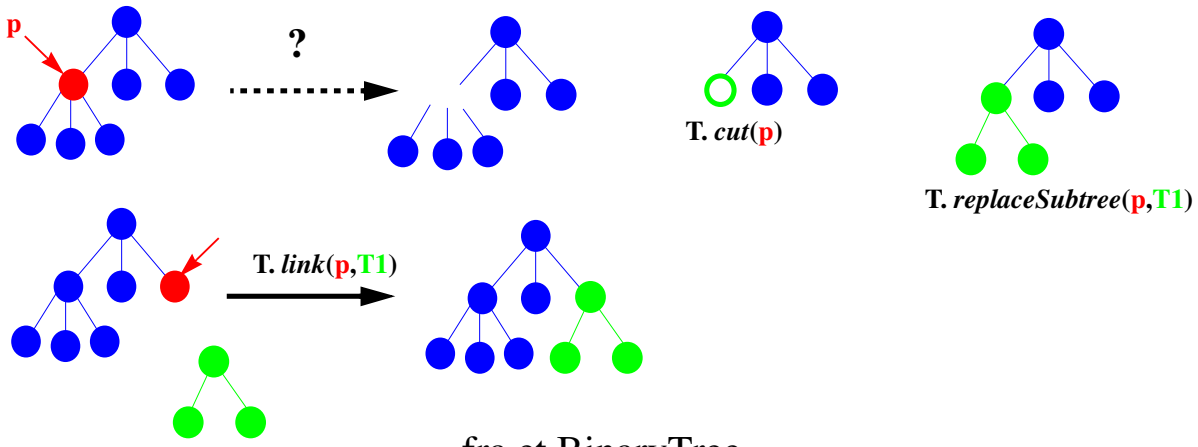
```
/**  Et tre har minst en ekstern node – rot */
public interface InspectableTree
        extends PositionalContainer {
    Position root();
    Position parent(Position v);
/**  Barn er ordnet fra venstre til høyre */
    Enumeration children(Position p);
    boolean isInternal(Position v);
    boolean isExternal(Position v);
    boolean isRoot(Position v);
    Enumeration siblings(Position p);
  }
```

```
public interface InspectableBinaryTree
        extends InspectableTree {
    Position leftChild(Position);
    Position rightChild(Position);
    Position sibling(Position);  }
```

```
public interface BinaryTree
        extends InspectableBinaryTree {
  Tree cut(Position) ;
  void link(Position, BinaryTree) ;
  Tree replaceSubtree(Position, BinaryTree);
  void expandExternal(Position) ;
  void removeAboveExternal(Position) ;  }
```

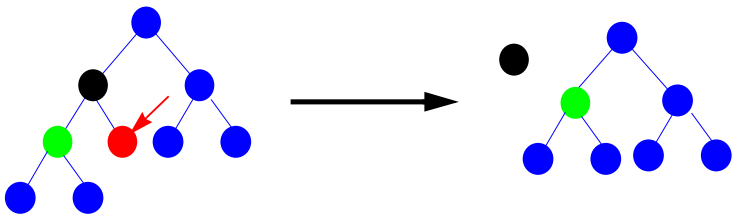
```
public interface Tree extends InspectableTree {
/**  Posisjon p er erstattet med en ny ekstern node
 *   med null element. */
    Tree cut(Position p);
/**  Posisjon p (må være ekstern) er erstattet med treet t */
    void link(Positionp p, Tree t);
/**  Erstatt subtre ved p med et nytt subtre t */
    Tree replaceSubtree(Position p, Tree t)     }
```


Bygging av trær

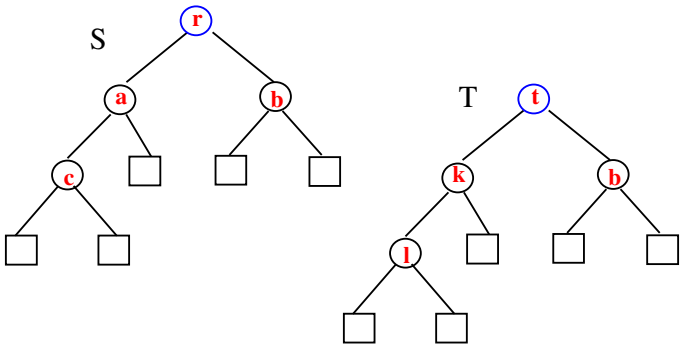


fra et BinaryTree

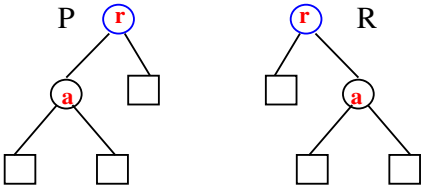
void *removeAboveExternal*(Position p)



BinaryTree: likhet vs. isomorfisme



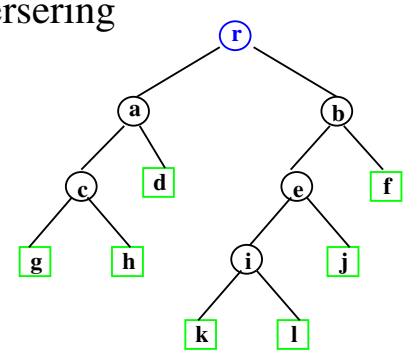

```
boolean iso (InspectableBinaryTree P)
{   return iso(root, P.root(), P) ;  }

boolean equals (InspectableBinaryTree P)
{   return equ(root, P.root(), P) ;  }

boolean iso // equ
        (Position r, Position p, InspectableBinaryTree P)
{   if (isExternal(r) && P.isExternal(p))
      return true; // r.element().equals(p.element());
    else if (isInternal(r) && P.isInternal(p) )
      return ( // r.element().equals(p.element())  &&
          iso(leftChild(r), P.leftChild(p), P) &&          // equ
          iso(rightChild(r), P.rightChild(p), P) );        // equ
    else return false;
}
```

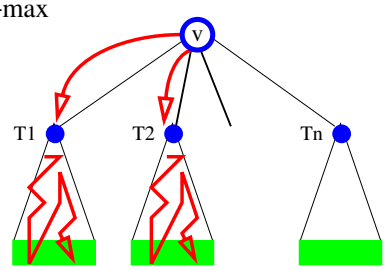


Tre-Algoritmer : traversering

Enumeration positions() – *ok ... men i hvilken rekkefølge ?*

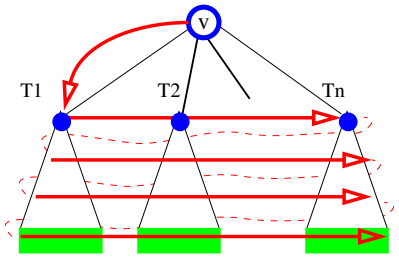
```
int height(Position v)
  if (isExternal(v)) return 0
  else // 1+max{ height(p): p i children(v) }
     max=0
     for hver p i children(v)
        h=height(p); if (h>max) max=h
     return 1+max
```






```
void DFS(Tree T, Position v) {
  for hver p i T.children(v)
      DFS(T,p)    }
```

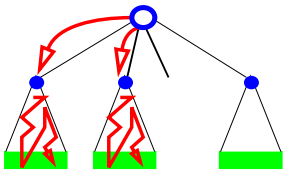
enumererer: **r, a,c,g,h,d, b, e,i,k,l,j, f**

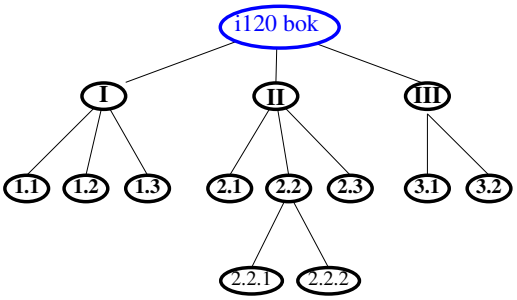


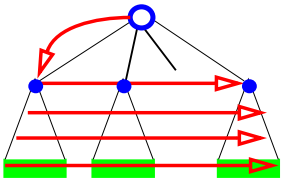
void **BFS**(Tree T, Position v)

enumererer: **r, a,b, c,d,e,f, g,h,i,j, k,l**

DFS vs. BFS







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- **1.2** OO-Programming
- **1.3** JAVA

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  - **2.2.2** Avarage Case
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- **3.2** Queues

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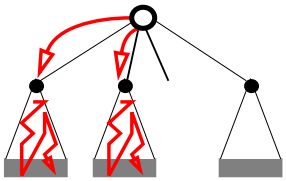
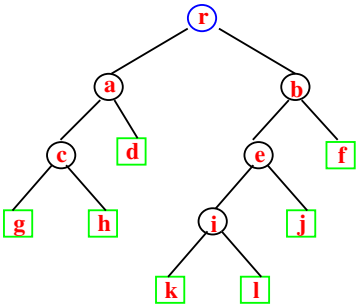
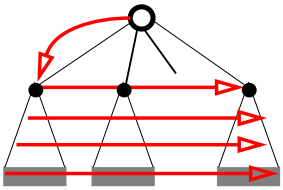
DFS og BFS

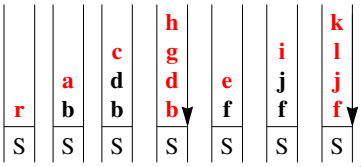






r, a,c, h,g, d, b, e,i,k,l, j, f

```
/*DFS(Tree T)
 *   Stack S = new StackIm()
 *   S.push(T.root())
 *   while (!S.isEmpty())
 *      p= S.pop()
 *      S.push(T.children(p))
 */
```



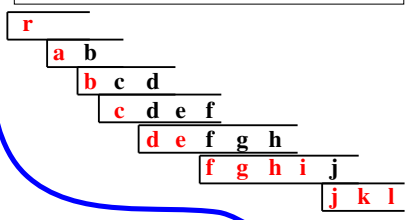
trav(T, new **StackIm**())

***"samme"** algoritme:*

trav(Tree T, **LiFi** S)
- S.**add**(T.root());
- while (! S.isEmpty())
  - p = S.**remove**();
  - S.**add**(T.children(p))

r, a,b, c,d,e,f, g,h,i,j, k,l

```
/*BFS(Tree T)
 *   Queue S = new QueueIm()
 *   S.enqueue(T.root())
 *   while (!S.isEmpty())
 *      p= S.dequeue()
 *      S.enqueue(T.children(p))
 */
```



trav(T, new **QueueIm**())

PreOrder

gjør jobben
FØR
rekursive kall

```
DFS(Tree T, Position v)
  Print(v.element())
  for hver p i T.children(v)
     DFS(T,p)
```

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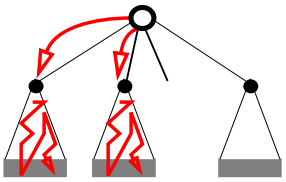
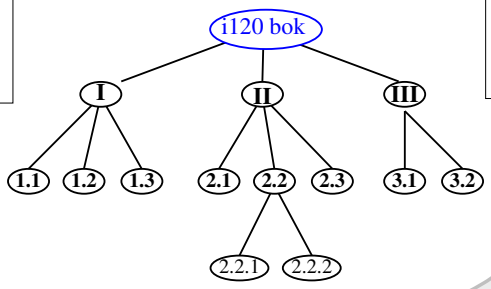
DFS






```
int du(Tree T, Position v)
  int v_du = 0 // lokal variabel
  for hver p i T.children(v)
     v_du= v_du + du(T,p)
( (Fil) v.element() ).size = v_du
```

PostOrder

gjør jobben
ETTER
rekursive kall

```
DFS(Tree T, Position v)

  for hver p i T.children(v)
     DFS(T,p)
  Print(v.element())
```

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Chap. III Basic DS

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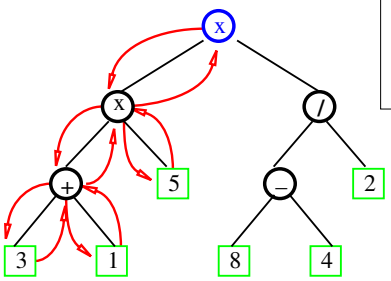
Binære Trær : **Euler-Tour & InOrder**

```
DFS(Tree T, Position v)
  ??? - pre
  for hver p i T.children(v)
     DFS(T,p)
  ??? - post
```

```
DFS(BinTree B, Position v)
  ??? – pre (left)
  if (isInternal(v))
     DFS(B,B.leftChild(v))
  ??? – inn (below)
  if (isInternal(v))
     DFS(B,B.rightChild(v))
  ??? – post (right)
```

```
DFS(BinTree B, Position v)
  if (B.isExternal(v)) ??? – ekst
  else
     ??? – pre
     DFS(B,B.leftChild(v))
     ??? – inn
     DFS(B,B.rightChild(v))
     ??? – post
```



((3 + 1) x 5) x ((8 – 4) / 2) = 40

```
void Pr(BinTree B, Position v)
  if (B.isExternal(v)) print(v.element())
  else
     print("(")
     Pr(B,B.leftChild(v))
     print(v.element())
     Pr(B,B.rightChild(v))
     print(")")
```

```
print(v.element())
Pr(B,B.leftChild(v))
Pr(B,B.rightChild(v))
```

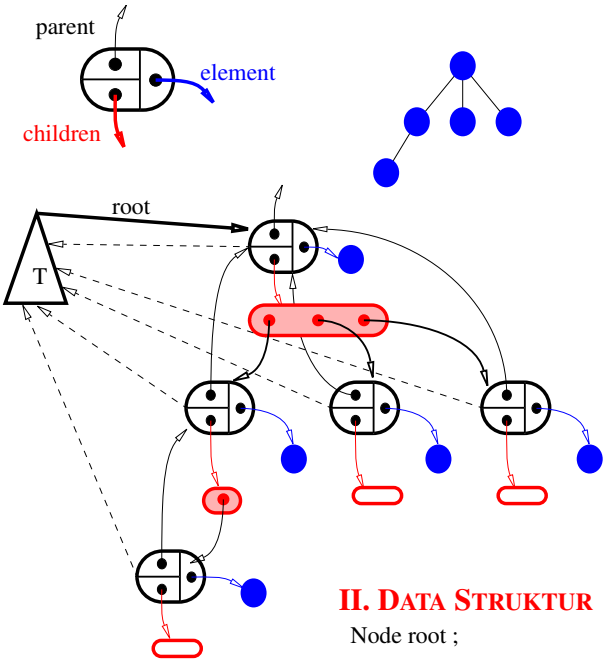
```
int val(BinTree B, Position v)
  if (B.isExternal(v))
    return ((Integer)v.element()).intValue()
  else
    L= val(B,B.leftChild(v))
    R= val(B,B.rightChild(v))
    return ((Oper)v.element()).op(L,R)
```

x x + 3 1 5 / – 8 4 2

x [x(+ (3,1), 5), / (–(8,4),2)]

Implementasjon av Tree – *med LenketStruktur*

I. Data Representasjon



II. Data Struktur

Node root ;

III. Data Invariant *: for alle Position* p, v *i* T:

- p.container() == T
- T.root() != null
- isRoot(p) ↔ (p.parent()==null)
- T.isEmpty() ↔ T.root().elem() == null
- isExternal(p) == p.children().isEmpty()
- isInternal(p) == !p.children().isEmpty()
- v.parent()==p ↔ v er bland p.children()
- p.children() != null

```
public class Node implements Position
{ private Object elem; private Node parent;
  private Container  cont;  private Sequence children;
  public Node(Object e, Node p, Container c) {
    setElement(e); setParent(p);  setContainer(c);
    children= new SequenceLL(); }
  public Object element() { return elem; }
  public void setElement(Object e) { elem= e; }
  public Container container() { return cont;}
  public void setContainer(Container c) { cont= c;
    for alle x i children() : x.setContainer(c); }
  public Node parent() { return parent; }
  public void setParent(Node p) { parent= p; }
  public Container children() { return children; }
  public void setChildren(Container c) { children= c; }
```

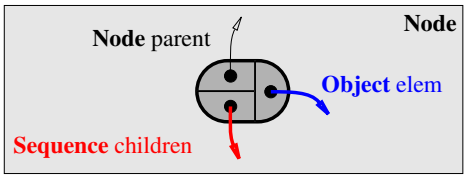

LenketStruktur implementasjon av Tree ADT

```
public class TreeLL implements Tree {
  private Node root; private int size;

  public TreeLL(Node n) {  size= 1; root= n;
    n.setParent(null); n.setContainer(this); }

  public TreeLL(Object e) {
    this(new Node(e, null, this)); }

  public Position root() { return root; }

  public Position parent(Position v) throws ... {
    return ok(v).parent(); }

  public Enumeration children(Position v) throws.. {
    gjør om ok(v).children() til Enumeration }
  //ikke ok(v).children().elements();}

  public boolean isInternal(Position v) throws ... {
    return !ok(v).children().isEmpty());}

  public boolean isExternal(Position v) throws ... {
    return ok(v).children().isEmpty());}

  public boolean isRoot(Position v) throws ... {
    return (v == root); }

  public int size() { return size; }

  public Object replace(Position v, Object e) throws..{
      Object tmp = ok(v).element();
      p.setElement(e); return tmp;  }

  public Tree cut(Position v) throws ... {
    Node n= ok(v);
    TreeLL ret= new TreeLL(n.element());
    Node retRoot = (Node)ret.root();
    Enumeration ch= children(n);
    while (ch.hasMoreElements()) {
      ((Node)ch.nextElement()).setParent(retRoot); }
    retRoot.setChildren(n.children());
    retRoot.setContainer(ret);
    n.setElement(null);  n.setChildren(null);
    size = size – ret.size() + 1;
    return ret; }  // v er fortsatt Position i dette treet

  private Node ok(Position p) throws InvalidPosExc {
    if ( p==null || p.container() != this ||
        !(p instanceof Node) ) throw new InvalidPosExc();
    return (Node)p; }

  public void link(Position v, Tree T) throws ... {
    // lettest når T også er TreeLL . . .
```

Implementasjon av BinaryTree ADT (FDT)

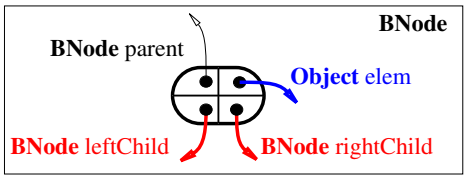
I. Utvid klassen TreeLL slik at:

- Container children() *alltid har 0 eller 2 elementer – pass på* link(p,T), replaceSubtree(p,T)
- *implementer* Position leftChild(), Position rightChild() – *skill i* children()




```
public class BNode implements Position
{ private Object elem; private Container cont;
  private BNode parent, left, right;
  public BNode(Object e, BNode p, Container c) {
    setElement(e); setParent(p); setContainer(c); }
  public Object element() { return elem; }
  . . .
  protected void setContainer(Container c) { cont = c;
    if (left!=null) left.setContainer(c);
    if (right!=null) right.setContainer(c) }
  public BNode leftChild() { return left; }
  protected void setLeft(BNode p) {
    left= p; p.setParent(this); p.setContainer(c); }
  public BNode rightChild() { return right; }
  protected void setRight(BNode p) {
    right= p; p.setParent(this); p.setContainer(c); }
```

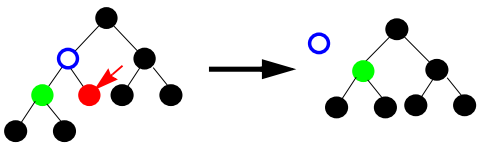
II. Bruk en annen BNode



LenketStruktur implementasjon av BinaryTree ADT

```
public class BTreeLL implements BinTree {
  private Position root;
  public BTreeLL(Object e) {
    root = new BNode(e, null, this);  }
  public Position parent(Position v) throws ... {
    return ok(v).parent();   }
  public Position leftChild(Position v) throws ... {
    return ok(v).leftChild();   }
  public Position setLeft(Position v, Object o) throws...{
    BNode n= ok(v);
    if (isExternal(n)) expandExternal(n);
    n.setLeft( new BNode(o, n, this) );
    return n.leftChild();   }
  public Position rightChild(Position v) throws..{ ... }
  public Position setRight(Position v, Object c) throws..{.}
  private BNode ok(Position p) throws InvalidPosExc {
    if ( p==null || p.container() != this ||
       !(p instanceof BNode) ) throw new InvalidPosExc();
    return (BNode)p;    }
  public int size() { return size(root); }
  private int size(Position p) { if (isExternal(p)) return 1;
    else return size(leftChild(p)) + size(rightChild(p)) +1; }
```

```
  public boolean isInternal(Position v) throws ... {
    BNode n= ok(v);
    return n.leftChild()!=null || n.rightChild()!=null);  }
  public boolean isExternal(Position v) throws ... {
    return ! isInternal(v) ;   }
  public void expandExternal(Position v) {
    BNode n= ok(v);
    if (isExternal(n)) {
      n.setLeft( new BNode(null, n, this) );
      n.setRight( newBNode(null, n, this) ); }    }
  public Object removeAboveExternal(Position v) throws...{
    BNode n = ok(v); Object o = v.element();
    if ( isInternal(n) || isRoot(n) ) throw ...
    BNode par = ok(parent(n)) ;
    BNode sib = ok(sibling(n));
    par.setElement(sib.element());
    par.setLeft(sib.leftChild());
    par.setRight(sib.rightChild());
    return o;
}
```



alt unntatt positions(), elements(), size() *er* O(1)

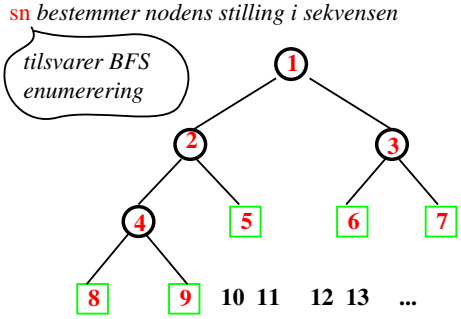
Sequence implementasjon av BinaryTree ADT

I. Data Representasjon

for en Position v i treet T, la sn(v) være et tall gitt ved:

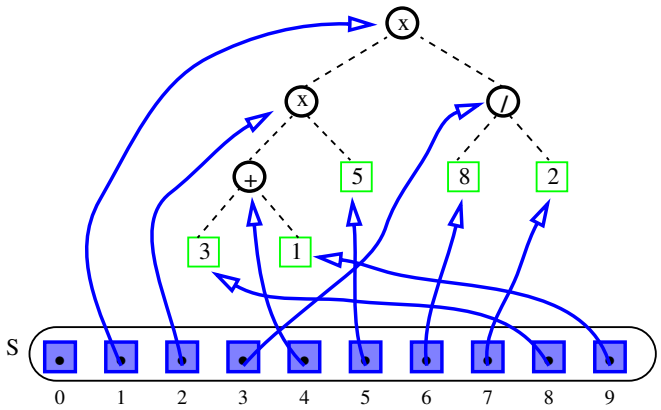
- hvis T.isRoot(v) så sn(v) = 1
- hvis T.leftChild(v) == u så sn(u) = 2*sn(v)
- hvis T.rightChild(v) == u så sn(u) = 2*sn(v) + 1

sn *bestemmer nodens stilling i sekvensen*



II. Data Struktur

Sequence S // *Sekvens-Position er Tree-Position*




```
Position leftChild(Position p) {
    return S . atRank( S.rankOf(p)*2 );}
Position root() { return S . atRank(1); }
```

- *alle* BinaryTree *operasjoner (unntatt positions(), elements()) kan fåes med* **O**(1) ***relativt*** *til* Sequence
- *Spesielt adekvat for "**komplette**" binære trær*

III. DataInvariant :

- S.atRank[1] == root
- S.atRank[2*v] == T.leftChild(S.atRank[v])
- S.atRank[2*v+1] == T.rightChild(S.atRank[v])

Ordboksøking (FDT)

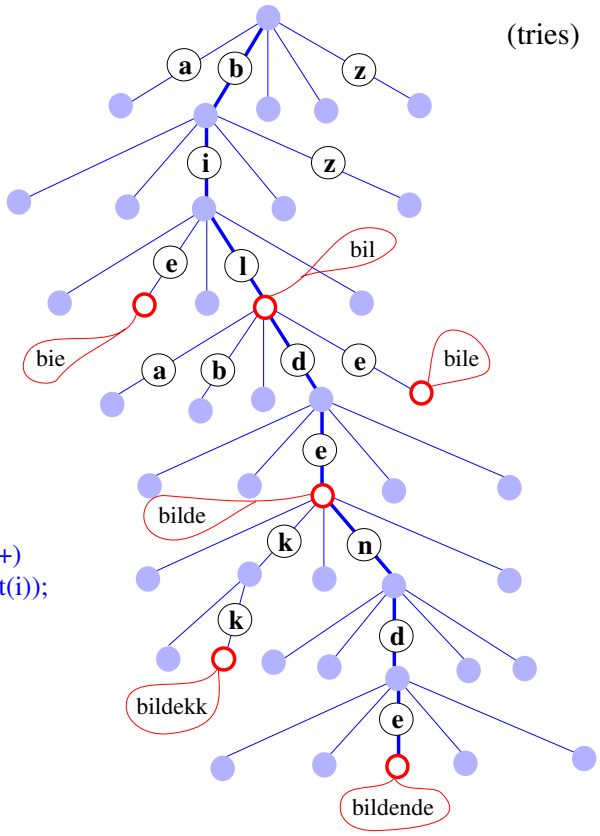
(tries)

...
bie
bil
bilateral
bilbelte
bilde
bildekk
bildende
bile
....

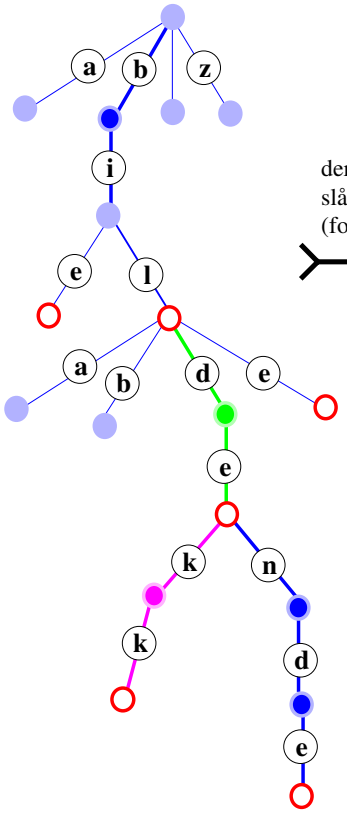
finn(String X)
= **O**(n)
ev. **O**(log n)

```
Position finn(String X)
    p = root;
    for (int i=0; i<X.length;i++)
        p = p.childNo(X.charAt(i));
        if (p == null) break;
    if (p == null || !p.isRed())
        finnes-ikke
    else return p;
```

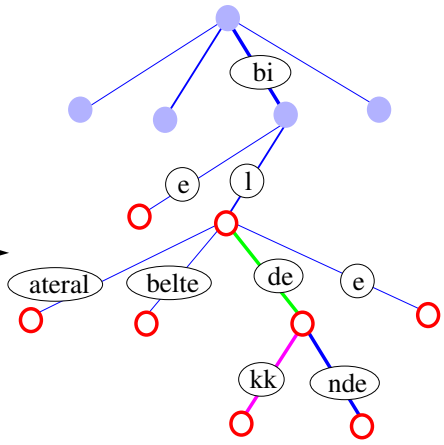
= **O**(X.length)



Komprimering



dersom node **n** har bare 1 barn **b**,
slå **n** og **b** sammen til 1 ny node
(fortsett nedover treet)




```
Position finn(String X, Position p)     // finn(S,root)
   if (X==e && p.isRed()) return p
   else if (X==e) return null
   else if (p.isExternal()) return null
   else  Postion r= null
      for (hver kant e = (p,w))
         t = e.string(); l = t.length;
         if (X.startsWith(t))
            r= finn(X[l+1...X.length], w); break ;
      return r;
```

= **O**(X.length)

Streng komprimering

iterer gjennom hele teksten S:

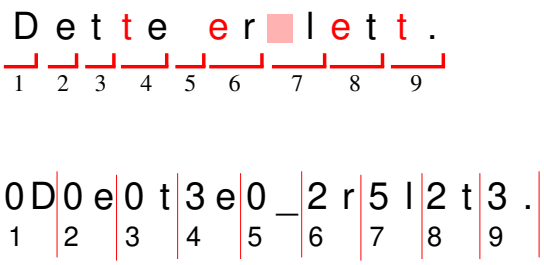
S = S[0...*i-1*]+ S[*i*...length]
C + *X*

p = lengste prefiks av ***X*** som er komprimert i ***C*** som ***cp***

t = tegnet i ***X*** rett etter ***p***

· utvid ***cp*** med **+*t***

· fortsett med ***i*** rett etter ***t*** i ***X***

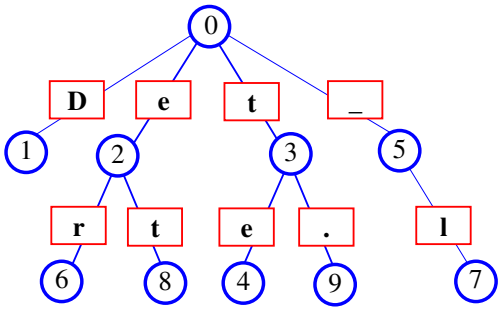


D e t t e e r l e t t .

D e t t e e r l e t t .

komprimert: ***C*** ***X*** : resten

i ***p*** ***t***



Oppsummering

1. Trær og Binarære Trær:

- *definisjoner og terminologi*
- *egenskaper*

2. Tre-algoritmer – traversering:

- *DFS og BFS*
- *DFS :*
  - *pre– og postorder,*
  - *innorder for BinaryTree*

3. Tree og BinaryTree ADT

4. Implementasjon av trær:

- *LenketStruktur*
- *Sekvens – BinaryTree*
- *kompleksitet*