# I220, Autumn 2002: Exercise 4

1. A group is a structure with one sort and a binary associative operation with identity and inverse. We may specify groups in the logic $EL$ by the signature

$$\begin{aligned} \Sigma = (\{S\}, \quad \{ & \_\circ\_ : S \times S \to S, \\ & 0 : \quad \to S, \\ & \_^{-} : S \to S \ \}) \end{aligned}$$

   and axioms (sentences, formulas) $\Phi \subseteq EL(\Sigma)$,

$$\begin{aligned} \Phi = \{ \ \forall \ \{x : S, y : S, z : S\} \ . \ & x \circ (y \circ z) = (x \circ y) \circ z, \\ \forall \ \{x : S\} \ . \ & 0 \circ x = x, \\ \forall \ \{x : S\} \ . \ & x \circ 0 = x, \\ \forall \ \{x : S\} \ . \ & x \circ (x^{-}) = 0, \\ \forall \ \{x : S\} \ . \ & (x^{-}) \circ x = 0 \ \}. \end{aligned}$$

   Show that for any group $G \in \mathbf{Mod}(\Phi)$ the equations $\Psi \subseteq EL(\Sigma)$ hold, where

$$\begin{aligned} \Psi = \{ \ \forall \ \{x : S\} \ . \ & (x^{-})^{-} = x, \\ \forall \ \{x : S, y : S\} \ . \ & (x \circ y)^{-} = y^{-} \circ x^{-} \ \}. \end{aligned}$$

   In other words, show that $\Phi \models \Psi$.

2. From the book: 5.2-8, 5.2-9, 5.2-10, 5.2-11.

3. From the book: 5.3-4 (i).

4. From the book: 5.4-1, 5.4-3, 5.4-5.

5. From the book: 5.5-2, 5.5-3.