New Results on the Time Complexity and Approximation Ratio of the Broadcast Incremental Power Algorithm

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Abstract

The Broadcast Incremental Power (BIP) algorithm is the most frequently cited method for the minimum energy broadcast routing problem. A recent survey concluded that BIP has $O(|V|^3)$ time complexity, and that its approximation ratio is at least 4.33. We strengthen these results to $O(|A|+|V|\log|V|)$ and 4.598, respectively.

Key words: Approximation Algorithms, Analysis of Algorithms, Wireless Ad hoc Network, Minimum Energy Broadcast.

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1 Introduction

In many applications of wireless ad hoc systems, a minimum energy broadcast routing has to be computed repeatedly and quickly. To establish a broadcast routing, a transmission power must be assigned to each network unit. The power needed to cover a set of receiving units is the maximum of the power needed to reach any of them, and grows at least quadratically with the distance to the receiving unit. Consequently, computing a minimum energy routing is NP-hard [2]. Therefore, the energy efficiency of applications depends on efficient routing heuristics. The Minimum Energy Broadcast Problem (MEBP) has attracted intensive research, of which an overview can be found in the survey of Guo and Yang [4]. The most frequently cited algorithm for MEBP is the Broadcast Incremental Power (BIP) algorithm by Wieselthier et al. [8].

Previous works on MEBP heuristics emphasize low time complexity and approximability [2], [6], [7]. In [8], Wieselthier et al. gave an implementation of BIP with $O(|V|^3)$ time complexity when applied to a graph $G = (V, A)$ representing a wireless network. In this work, we present an implementation having $O(|A| + |V| \log |V|)$ time complexity by adapting the $O(|A| + |V| \log |V|)$ implementation of Prim's algorithm using Fibonacci heaps.

Wan et al. [6] together with Klasing et al. [5] showed that, under assumptions commonly made on wireless signal propagation, the approximation ratio of BIP is between $4 + \frac{1}{3}$ and 12.15. Due to a lemma in [6] and Ambühl's work [1], the currently best upper bound on the approximation ratio of BIP is 6. Retaining the assumptions in [5] and [6], we strengthen the lower bound by giving a sequence of MEBP instances for which the optimal power consumption decreases towards 1, and for which the power consumption of BIP's solution increases beyond 4.598. Worst-case instances do not only provide lower bounds on the approximation ratio, but also point out an algorithm's weakness, and thus suggest directions for future algorithm development.

2 Preliminaries

A problem instance is given by a graph $G = (V, A)$, a source $s \in V$, and power requirements (costs) $c : A \mapsto \mathbb{R}$. The nodes and arcs represent the networking units and potential wireless links, respectively.

A solution can be given by an s-arborescence $T = (V_T, A_T)$ with node set $V_T \subseteq V$ and arc set $A_T \subseteq A$. An s-arborescence is a directed tree where all arcs are oriented away from s. An s-arborescence T induces for every v a power assignment $p_v(T)$, which either is 0 or the cost c_{vw} of the most expensive arc (v, w) leaving v in A_T. Thereby, the cost of T is $p_T = \sum_{v \in V_T} p_v(T)$, and the minimum energy broadcast problem can be formulated as

[**MEBP**] Find an s-arborescence T such that $V_T = V$ and p_T is minimized.

BIP constructs an s-arborescence $T = (V_T, A_T)$ in a way similar to Prim's construction of a minimum spanning tree. Starting from $T = (\{s\}, \emptyset)$, BIP evaluates all arcs (u, v) where $u \in V_T$ and $v \notin V_T$ by the incremental power $c_{uv} - p_u(T)$. An arc (u, v) minimizing this difference is selected, and v and (u, v) are added to T. This is repeated until T spans V.

3 Improved Time Complexity of BIP

Our implementation (Tab. 1) follows the implementation of Prim's algorithm in [3]. It keeps all vertices $v \in V \setminus V_T$ in a min-priority queue Q based on a *key* field containing the minimum incremental cost of adding v to V_T. The field $\pi[v]$ contains a node in V_T to which v can be linked at cost $key[v]$. In Tab. 1, V_T and A_T are represented by $V \setminus Q$ and $\{(\pi[v], v) : v \in V_T \setminus \{s\}\}$, respectively. The adjacency list $Adj[v]$ contains all nodes w for which $(v, w) \in A$.

Table 1. Implementation of BIP

BIP$(G = (V, A), s, c)$

1 **for all** $v \in V \setminus \{s\}$

2   $key[v] \leftarrow c_{sv}$

3   $\pi[v] \leftarrow s$

4 *priority queue* $Q \leftarrow V \setminus \{s\}$

5 **while** $Q \neq \emptyset$

6   $v \leftarrow \text{extractMin}(Q)$

7   **for all** $w \in Adj[v] \cap Q$

8   **if** $c_{vw} < key[w]$

9       $key[w] \leftarrow c_{vw}$

10       $\pi[w] \leftarrow v$

11   **for all** $w \in Adj[\pi[v]] \cap Q$

12   **if** $(c_{\pi[v]w} - c_{\pi[v]v} < key[w])$

13       $key[w] \leftarrow c_{\pi[v]w} - c_{\pi[v]v}$

14       $\pi[w] \leftarrow \pi[v]$

15 **return** $T = (V, \{(\pi[v], v) :$ $v \in V \setminus \{s\}\})$

Theorem 1 *The implementation in Tab. 1 has* $O(|A| + |V| \log |V|)$ *time complexity.*

PROOF. Omitting Steps 11-14 in Tab. 1 gives an implementation of Prim's algorithm, that, if Q is implemented as a Fibonacci heap, has $O(|A| + |V| \log |V|)$ time complexity [3]. Steps 11-14 contain $O(|A|)$ assignments and *key*-update operations (Step 13). With Q implemented as a Fibonacci heap, these run in constant amortized time. Thus Steps 11-14 have $O(|A|)$ time complexity, and the theorem follows. □

For determining a multicast routing that reaches all nodes in a specified destination set, Wieselthier et al. suggested in [7] to apply BIP first, and then omit ("prune") all arcs not leading to a destination, resulting in the Multicast

Incremental Power (MIP) algorithm. Pruning is done by traversing the arborescence upwards from the leaves to the first node that either is the source, or a destination, or a node with more than one child. Traversed arcs and their head nodes are deleted. Next, $p_v(T)$ ($\forall v \in V_T$) can be computed using breadth first search (BFS) in T. Since both traversing T and BFS have $O(|V|)$ time complexity, MIP has $O(|A| + |V| \log |V|)$ time complexity.

4 A New Lower Bound on BIP's Approximation Ratio

For any u, v, $w \in \mathbb{R}^2$ and $r \in \mathbb{R}$, we denote by

- uv the line segment with end points u and v,
- d_{uv} the length of uv, i.e. the Euclidean distance between u and v,
- $\angle uvw$ the angle between the line segments uv and vw with positive (counterclockwise) direction from uv to vw, that is, the angle for which

$$\cos \angle uvw = \frac{d_{uv}^2 + d_{vw}^2 - d_{wu}^2}{2 d_{uv} d_{vw}}, \tag{1}$$

- $C(u, r) = \{x \in \mathbb{R}^2 : d_{ux}^2 = r^2\}$ = the circle with radius r centered at u.

An important evaluation criterion for algorithms is the performance of the algorithm solution relative to the optimal one. For any instance $\mathcal{I}$ of a minimization problem and any algorithm $\mathcal{A}$, the *performance ratio* $\rho_{\mathcal{A}}(\mathcal{I})$ is defined as the cost of the algorithm's solution divided by the cost of the optimal solution. The supremum $\sup_{\mathcal{I}} \rho_{\mathcal{A}}(\mathcal{I})$ over all possible input instances is called the *approximation ratio* of $\mathcal{A}$, on which a lower bound is given by $\rho_{\mathcal{A}}(\mathcal{I})$ for any instance $\mathcal{I}$.

It follows from [1] and [6] that $\sup_{\mathcal{I}} \rho_{\text{BIP}}(\mathcal{I}) \in \left[4 + \frac{1}{3}, 6\right]$, if the supremum is taken over instances where G is complete, V is a finite set of points in $\mathbb{R}^2$, and c_{uv} is proportional to d_{uv}^2 $\forall (u,v) \in A$. Since the proportional factor is irrelevant in our analysis we assume $c_{uv} = d_{uv}^2$ $\forall (u,v) \in A$. For this type of instances, we construct a sequence for which the performance ratio of BIP increases beyond 4.598.

4.1 The best lower bound known from the literature

In [6], Wan et al. gave an instance for which BIP outputs an s-arborescence with power consumption arbitrarily close to $4 + \frac{1}{3}$ times the optimal. To the best of our knowledge, this is the best lower bound on the approximation ratio of BIP known to date. In Fig. 1, we depict a slightly modified version of the instance in [6], yielding the same bound. The modification is made in order to prepare for a stronger bound, which through extensions of the instance in Fig. 1 will be derived in subsequent sections.

Our instance contains the nodes $a_0, \ldots, a_4, b_0, \ldots, b_3$, and $z_0, \ldots, z_m$, where $m > 1$ is an integer, $a_j = \left(\cos\frac{(2+j)\pi}{3}, \sin\frac{(2+j)\pi}{3}\right)$ $(j = 0, \ldots, 4)$, $b_j = \left(1 + \frac{2}{\sqrt{3}m}\right) a_j$ $(j = 0, \ldots, 3)$, $z_m = \left(0, \frac{1}{\sqrt{3}}\right)$, and $z_j = \frac{j z_m}{m}$ $(j = 0, 1, \ldots, m)$. Hence, $z_0 = (0,0)$, and $z_0, \ldots, z_m$ are uniformly distributed along $z_0 z_m$, and $a_0, \ldots, a_4$ $(b_0, \ldots, b_3)$ are positioned on (close to) the unit circle $C(z_0, 1)$. We let the source be $s = z_m$.

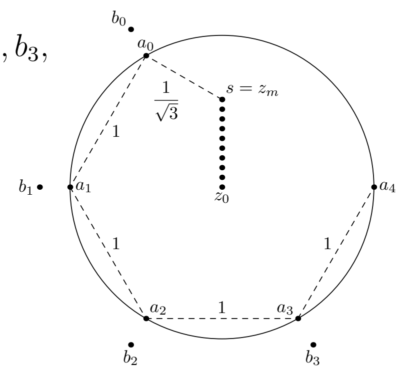

Fig. 1. Instance where BIP has performance ratio $4 + \frac{1}{3}$

The idea in [6] is to make BIP prefer chordal arcs (e.g. (a_0, a_1)) to the corresponding radial arcs (e.g. (z_0, a_1)), which in [6] is accomplished by perturbing the position of some of the nodes on $C(z_0, 1)$ such that d_{a_{j-1},a_j} becomes marginally smaller than $d_{z_0 a_j}$. We apply the same idea, and the desired effect is obtained by encouraging a marginal power assigned to $a_0, \ldots, a_3$ in order to reach $b_0, \ldots, b_3$ (not present in the instance of [6]), respectively.

It is readily seen that for the instance in Fig. 1 with sufficiently large m, some optimal arborescence contains arcs $(z_m, z_{m-1}), \ldots, (z_1, z_0)$, (z_m, a_0), $(a_0, b_0), \ldots, (a_3, b_3)$, $(z_0, a_1), \ldots, (z_0, a_4)$, resulting in a power consumption of $1 + O(m^{-1})$. The arborescence T' produced by BIP contains the arcs $(z_m, z_{m-1}), \ldots, (z_1, z_0)$, (z_m, a_0), $(a_0, b_0), \ldots, (a_3, b_3)$, $(a_0, a_1), \ldots, (a_3, a_4)$, resulting in $p_{T'} = 4 + \frac{1}{3} + O(m^{-1})$.

The three terms of $p_{T'}$ reflect the path $(a_0, \ldots, a_4)$, the arc connecting a_0 to the source, and arcs of marginal length, respectively. In the following, a better lower bound on the approximation ratio of BIP is derived from similar instances, where:

- the optimal power consumption remains close to 1,
- BIP produces an arborescence containing the path $(a_0, \ldots, a_4)$, and
- BIP connects a_0 to the source at a cost higher than $\frac{1}{3}$.

4.2 Increasing the cost of connecting a_0 to the source

Let the positions of nodes $z_0, a_0, \ldots, a_4$ be the same as in Sect. 4.1. In order to increase the cost of connecting a_0, we define a new source s_1 in a position further away from a_0. In the new instance $\mathcal{I}_1$ shown in Fig. 2, we also:

- Maintain a set Z_1 of $m+1$ nodes uniformly distributed along some curve ζ_1 with end points s_1 and z_0, thus keeping the optimal power consumption arbitrarily close to 1.
- Keep a_3 as a closest neighbor to a_4, so that BIP links a_4 to a_3.
- Keep a_0 at least as close to s_1 as to any other node in Z_1, so that BIP links a_0 to s_1.

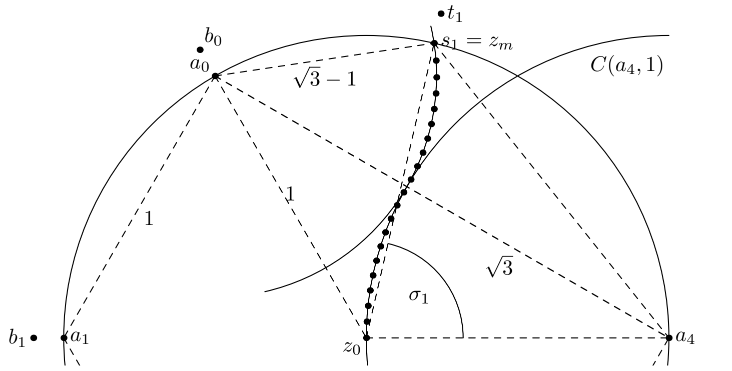

Fig. 2. Instance $\mathcal{I}_1$ with performance ratio $\rho_{\text{BIP}}(\mathcal{I}_1) > 4 + \frac{1}{3}$

The two last restrictions imply $d_{za_0} \geq d_{s_1 a_0}$ and $d_{za_4} \geq 1$ for all $z \in Z_1$, meaning that ζ_1 cannot intersect the interiors of $C(a_0, d_{a_0 s_1})$ and $C(a_4, 1)$. Hence we must choose s_1 close enough to a_0 to make the circles intersect at only one point, which is obtained by letting $d_{a_0 s_1} = d_{a_0 a_4} - 1 = \sqrt{3} - 1$. Among the two intersection points in $C(a_0, \sqrt{3}-1) \cap C(z_0, 1)$, we let the source s_1 be the point closer to a_4. We let ζ_1 be the curve which starts at s_1, follows $C(a_0, \sqrt{3}-1)$ in negative direction to the unique point of intersection of $C(a_0, \sqrt{3}-1)$ and $C(a_4, 1)$, and from there follows $C(a_4, 1)$ in positive direction to z_0. Let $z_m = s_1$, and let the set Z_1 consist of $m+1$ nodes $z_m, \ldots, z_0$ distributed along ζ_1 such that $d_{z_m z_{m-1}} = \cdots = d_{z_1 z_0} = \varepsilon_1$, implying that ε_1 tends to 0 as m grows towards infinity. The node set of instance $\mathcal{I}_1$ is $S_1 = Z_1 \cup \{a_0, \ldots, a_4, b_0, \ldots, b_3, t_1\}$, where $b_j = (1 + 2\varepsilon_1)a_j$, $(j = 0, \ldots, 3)$, and $t_1 = (1 + 2\varepsilon_1)s_1$.

Note that in order to reach a_0, the source must at least be assigned power $\left(\sqrt{3}-1\right)^2 = 4-2\sqrt{3}$. In the next section, we prove that BIP assigns this power to s_1, and that $\mathcal{I}_1$ yields an improved lower bound on the approximation ratio of BIP. However, an even stronger bound is achieved by generalizing the instance, and we therefore give the proof for a class of instances including $\mathcal{I}_1$.

4.3 Instances with performance ratio > 4.598

The construction of $\mathcal{I}_1$ indicates that Z_1 can be extended by adding nodes in the region bounded by $C(z_0, 1)$, $C(a_4, 1)$ and $C(a_0, \sqrt{3}-1)$. Our idea is to find a sequence of new source locations diverging from a_0 and converging to $C(a_4, 1)$, while satisfying the conditions given at the end of Sect. 4.1.

In the following, we construct a sequence $\{\mathcal{I}_1, \mathcal{I}_2, \ldots\}$ of MEBP-instances having the optimal power consumption converging to 1, and the power consumption of BIP's solution converging to a number larger than 4.598. Instance $\mathcal{I}_i$ is given by a recursive definition of a source s_i and a curve ζ_i with end points s_i and z_0. The basis of this recursion is s_1 and ζ_1 introduced in Fig. 2.

For convenient notation, let $s_0 = a_0$. Generalizing the determination of s_1, the location of s_i $(i \geq 1)$ is the intersection point in $C\left(s_{i-1}, d_{s_{i-1}a_4} - 1\right) \cap C\left(z_0, 1\right)$ closest to a_4 (Fig. 3). The curve ζ_i follows $C\left(s_{i-1}, d_{s_{i-1}s_i}\right)$ from s_i until it reaches ζ_{i-1}, after which ζ_i and ζ_{i-1} coincide. We let the set Z_i consist of nodes $z_m, \ldots, z_0$, where $z_m = s_i$, distributed along ζ_i such that $d_{z_m z_{m-1}} = \cdots = d_{z_1 z_0} = \varepsilon_i$, where m is sufficiently large to satisfy $2\varepsilon_i < d_{s_{i-1}s_i}$. To complete the definition of $\mathcal{I}_i$, let $b_j = a_j(1+2\varepsilon_i)$, $(j = 0, \ldots, 3)$, $t_j = s_j(1+2\varepsilon_i)$, $(j = 1, \ldots, i)$, and let the node set of $\mathcal{I}_i$ be $S_i = Z_i \cup \{a_0, \ldots, a_4, b_0, \ldots, b_3, s_1, \ldots, s_i, t_1, \ldots, t_i\}$.

We have $s_0 = (x_0, y_0) = \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$, and $s_i = (x_i, y_i)$ $(i = 1, 2, \ldots)$ is given by:

$$(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2 = \left(\sqrt{(x_{i-1} - 1)^2 + y_{i-1}^2} - 1\right)^2 \quad i = 1, 2, \ldots \tag{2}$$

$$x_i^2 + y_i^2 = 1 \quad i = 1, 2, \ldots \tag{3}$$

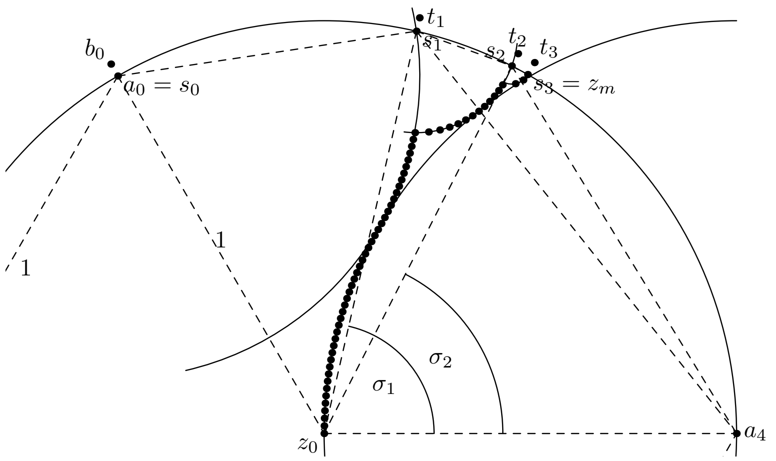

Fig. 3. Instance $\mathcal{I}_3$ with performance ratio $\rho_{\mathrm{BIP}}(\mathcal{I}_3) > 4.598$

```
1 for j ← m ... 1
2   link z_{j-1} to z_j
3 for j ← i ... 1
4   link t_j to s_j
5   link s_{j-1} to s_j
6 for j ← 0 ... 3
7   link b_j to a_j
8   link a_{j+1} to a_j
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Table 2. How BIP processes $\mathcal{I}_i$

Theorem 2 *The performance ratio $\rho_{BIP}(\mathcal{I}_i)$ is $4 + \sum_{j=1}^{i} c_{s_j s_{j-1}} + O\left(m^{-1}\right)$.*

PROOF.

The optimal total power is $1 + O\left(m^{-1}\right)$. We prove that BIP processes $\mathcal{I}_i$ as shown in Tab. 2, yielding total power consumption $4 + \sum_{j=1}^{i} c_{s_j s_{j-1}} + O\left(m^{-1}\right)$.

Since $2\varepsilon_i < d_{s_i s_{i-1}}$, Steps 2 and 4 are obvious. Step 5 follows from the fact that adding s_{j-1} to the tree gives minimum incremental cost, and s_j is the best choice for linking s_{j-1}.

After the execution of Steps 1-5, assume the for-loop 6-8 has been executed $j \in \{0, \ldots, 3\}$ times, which means that $a_1, \ldots, a_{j+1}$ and $b_0, \ldots, b_j$ have been added. It is then obvious that Step 7 follows since (a_j, b_j) is the only arc adding a new node to the tree with $O(m^{-2})$ incremental cost, for $j = 0, \ldots, 3$. If $j < 3$, Step 8 follows since the corresponding incremental cost is $1 - 4\varepsilon_i^2$, whereas all other options of tree augmentation cost at least $1-\varepsilon_i^2$. If $j = 3$, a_4 is the only node that is not yet reached. In order to show that it is linked to a_3 rather than any of $s_1, s_2, \ldots, s_i$, we prove $c_{s_j a_4} - c_{s_j s_{j-1}} > 1 \ \forall j = 1, \ldots, i$.

By applying (2)-(3) for $i = 1$ and $i = 2$, we obtain $(x_1, y_1) \approx (0.224, 0.975)$, $(x_2, y_2) \approx (0.455, 0.891)$, $c_{s_1 a_4} - c_{s_1 s_0} \approx 1.02 > 1$ and $c_{s_2 a_4} - c_{s_2 s_1} \approx 1.03 > 1$.

To prove $c_{s_i a_4} - c_{s_i s_{i-1}} > 1$ for $i > 2$, let $\sigma_i = \angle a_4 z_0 s_i$ $(i = 0, 1, \ldots)$, yielding $\angle a_4 s_{i-1} s_i = \frac{\sigma_i}{2}$ $(i > 0)$. As $d_{s_i a_4} > 1$ by construction, we have $\sigma_i > \frac{\pi}{3}$. Thus $\cos \angle a_4 s_{i-1} s_i < \frac{\sqrt{3}}{2}$. By (1),

$$\cos \angle a_4 s_{i-1} s_i = \frac{c_{s_{i-1} a_4} + c_{s_{i-1} s_i} - c_{s_i a_4}}{2 d_{s_{i-1} a_4} d_{s_{i-1} s_i}} \text{ . This yields}$$

$$\begin{aligned} &c_{s_i a_4} - c_{s_i s_{i-1}} = c_{s_{i-1} a_4} - 2\cos \angle a_4 s_{i-1} s_i \cdot d_{s_{i-1} a_4} d_{s_{i-1} s_i} > \\ &> c_{s_{i-1} a_4} - \sqrt{3} \cdot d_{s_{i-1} a_4} d_{s_{i-1} s_i} = \\ &= c_{s_{i-1} a_4} - \sqrt{3} \cdot d_{s_{i-1} a_4} \left(d_{s_{i-1} a_4} - 1 \right) = \left(1 - \sqrt{3} \right) c_{s_{i-1} a_4} + \sqrt{3} d_{s_{i-1} a_4}. \end{aligned}$$

The polynomial $(1 - \sqrt{3})x^2 + \sqrt{3}x - 1$ attains positive values between its zeroes, which are at $x = 1$ and $x \approx 1.366$. By construction, we have $1 < c_{s_i a_4} \leq c_{s_2 a_4} \approx 1.09$. Hence $c_{s_i a_4} - c_{s_i s_{i-1}} > 1$ $(i = 3, 4, \ldots)$, and Step 5 follows.

The proof is completed by observing that the output of Table 2 has a total power of $4 + \sum_{j=1}^{i} c_{s_j s_{j-1}} + O(m^{-1})$. $\square$

Corollary 3 *The approximation ratio of BIP is at least*
$$4 + \sum_{j=1}^{\infty} c_{s_j s_{j-1}} > 4 + \sum_{j=1}^{3} c_{s_j s_{j-1}} > 4.598.$$

PROOF. Solving (2)-(3) numerically for $i = 1, \ldots, 3$ gives the result. $\square$

A simple analysis shows that to a precision of three decimals, 4.598 is the best achievable bound by the sequence of instances. It is seen from Fig. 3 that $\sum_{j=i+1}^{\infty} c_{s_j s_{j-1}} < c_{s_i s_\infty}$, where $s_\infty = \lim_{i\to\infty} s_i = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Solving (2)-(3) also for $i = 4$ yields $4 + \sum_{j=1}^{4} c_{s_j s_{j-1}} = 4.5983 \pm 0.5 \cdot 10^{-5}$ and $c_{s_4 s_\infty} < 0.5 \cdot 10^{-5}$.

4.4 Extension to sweep

Wieselthier et al. [7] introduced the local search method *sweep* for MEBP. Given a spanning arborescence T, *sweep* searches for an arc $(v, w) \in A_T$ and a node $u \in V \setminus \{v, w\}$ such that $p_v(T) = c_{vw}$, $p_u(T) \geq c_{uw}$, and u is not a descendant of w in T. Such a combination implies that (v, w) can be replaced by (u, w) in A_T, thus reducing the power at v (unless $c_{vw} = c_{vw'}$ for some other child w' of v).

If T is the arborescence produced by applying BIP to $\mathcal{I}_i$, it is easily checked that for all feasible *sweep* moves, we have $(v, w) \in \{(z_m, z_{m-1}), \ldots, (z_1, z_0)\}$. Note that $(v, w) \in \{(s_i, s_{i-1}), \ldots, (s_2, s_1)\}$ with $u = a_0$ is infeasible since a_0 descends from $s_1, \ldots, s_{i-1}$. Since $p_{z_j}(T) = O\left(m^{-2}\right)$ $(j = 1, \ldots, m-1)$, the power reduction obtainable by *sweep* is only $O\left(m^{-1}\right)$. Thus Corollary 3 also applies when BIP and *sweep* are run sequentially.

5 Conclusions

We have proposed an implementation of BIP/MIP with $O(|A| + |V| \log |V|)$ time complexity, and demonstrated that the approximation ratio of BIP is larger than 4.598. The latter holds also if the local improvement method *sweep* is applied to the solution produced by BIP.

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