Improved Time Complexities of Algorithms for the Directional Minimum Energy Broadcast Problem

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Abstract. Ability to find a low-energy broadcast routing quickly is vital to a wireless system's energy efficiency. Directional antennae save power by concentrating the transmission towards the intended destinations. A routing is given by assigning a transmission power, angle, and direction to every networking unit, and the problem of finding such a power saving routing is called the Directional Minimum Energy Broadcast Problem (D-MEBP). In the well known Minimum Energy Broadcast Problem (MEBP), the transmission angle is fixed to 2π. Previous works suggested to adapt MEBP algorithms to D-MEBP by two procedures, Reduced Beam (RB) and Directional (D). As the running time of the routing algorithms is a critical factor, we reduce the time complexity of both by one order of magnitude.

1 Introduction

In wireless ad-hoc networks, a broadcast session is established without use of any central backbone system, and is based entirely on message passing between network units. To accomplish this, each unit is equipped with an energy resource in terms of a battery. Since this resource is limited it becomes crucial to route the broadcast messages in such a way that power consumption is minimized. At each network unit transmitting a message, the power consumption typically depends on the transmission coverage, which in its turn is determined by the set of intended recipients.

What parameters that can be set in order to achieve a minimum energy broadcast routing depends on the technology of the transmission antennae in the network. In the case of directional antennae, the transmission beam is concentrated towards the intended destination units, and the coverage hence has both a radial and an angular dimension. The former is simply the power required to reach the most remotely located recipient, and the latter is the minimum angle of a sector containing all. For networks based on omnidirectional antennae, the transmission angle is fixed to 2π.

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The Minimum Energy Broadcast Problem (MEBP) has in the case of omnidirectional antennae attracted intensive research. As the problem is NP-hard [1], the energy efficiency of applications depends on efficient routing heuristics. An overview of various suggestions to such methods can be found in the survey of Guo and Yang [2].

A common approach is to represent the network as a graph and determine a routing arborescence spanning the nodes in the graph. The arborescence defines an assignment of power to the nodes, given as the cost of the most expensive outgoing arc. A straightforward choice of routing arborescence is the Minimum Spanning Tree (MST), computed for instance by Prim's algorithm, which has been studied thoroughly. In [3], Guo and Yang proved that MST provides the optimal solution to a variant of MEBP, the static Maximum Lifetime Multicast Problem (MLMP).

Arborescences yielding a smaller total power assignment are found by taking into account the node-oriented objective function. In construction algorithms, where new nodes are added iteratively to an arborescence consisting initially of only the source node, this can be reflected by selecting nodes such that the incremental power is minimized. The most frequently cited such algorithm is the Broadcast Incremental Power (BIP) algorithm by Wieselthier et al. [4].

Assuming that the antennae are directional, we arrive at an extension of MEBP referred to as the Directional MEBP (D-MEBP). This problem has been studied to a far lesser extent than MEBP. Wieselthier et al. suggested in [5] the principles Reduced Beam (RB) and Directional (D) to adapt BIP to D-MEBP, resulting in the heuristics RB-BIP and D-BIP, respectively. RB-BIP first calls BIP to construct a broadcast routing arborescence, and then simply reduces the transmission angle of every unit to the minimum angle necessary to cover all the unit's children. D-BIP, on the other hand, takes antenna angles into account already in the construction phase. In each iteration of this procedure, the increase in both power requirement and angle are considered when deciding which unit to add to the current arborescence. In general, RB can be considered as a local improvement procedure to be called after construction, whereas D is interleaved with the construction algorithm.

In [6], Guo and Yang presented a mixed integer programming model, and used RB and D to adapt their local search heuristic [7] to D-MEBP. In [3], they applied both principles to adapt the MST algorithm to a directional version of MLMP.

As demonstrated in the above articles, RB and D are useful for adapting MEBP algorithms to D-MEBP in general. Wieselthier et al. showed in [8] that the time complexities of RB-BIP and D-BIP are $O(|V|^3)$ and $O(|V|^3 \log |V|)$, respectively, where V denotes the node set of the graph. The result for RB-BIP is derived from an implementation of BIP with $O(|V|^3)$ time complexity. The additional time complexity of the RB procedure is in [8] proved to be bounded by $O(|V|^2 \log |V|)$.

In this paper, we first improve the time complexity of RB to $O(|V| \log |V|)$ by better analysis. Together with an implementation of BIP with $O(|A|+|V| \log |V|)$

time complexity, suggested by Bauer et al. [9], this results in an implementation of RB-BIP with as low time complexity as $O(|A| + |V| \log |V|)$. Here A denotes the set of arcs in the graph.

Second, we suggest a novel implementation of D-BIP building on the BIP implementation in [9], and prove that its running time is $O(|V|^2)$.

2 Preliminaries

An instance of D-MEBP is given by a directed graph $G = (V, A)$, where the nodes represent the networking units, a source $s \in V$, power requirements $c \in \mathbb{R}^A$, and the minimum transmission angle $\theta_{\min}$. The nodes are associated with points in the plane, and the power requirement c_{vu} is typically proportional to d_{vu}^{α}, where d_{vu} is the Euclidean distance between nodes v and u, and $\alpha \in [2, 4]$ is a constant [2].

A solution to any instance can be given by an s-arborescence $T = (V, A_T)$ with arc set $A_T \subseteq A$. An s-arborescence is a directed tree where all arcs are oriented away from s. In T, every node v has a (possibly empty) set $\Gamma_v(T)$ of children. The transmission power induced by T at $v \in V$ is given by

$$p_v(T) = \begin{cases} 0 & \text{if } \Gamma_v(T) = \emptyset \\ \max_{w \in \Gamma_v(T)} \{c_{vw}\} & \text{otherwise .} \end{cases}$$

In the idealized model assumed in the literature, the energy emitted by node v is concentrated uniformly in a beam of width $\theta_v(T)$ [5]. To simplify the definition of $\theta_v(T)$, nodes are identified with points in $\mathbb{R}^2$. For any two nodes u and v, we let uv denote the straight line segment in $\mathbb{R}^2$ with end points u and v, and for any three nodes u, v and w, we let $\angle_{uvw}$ denote the angle between the line segments uv and vw with positive (counter-clockwise) direction from uv to vw. This implies $\angle_{uvw} = 2\pi - \angle_{wvu}$. For the purpose of simplified presentation, we assume that no three nodes are collinear, and we define $\angle_{uvu} = 2\pi$. Let the *sector* S_{uvw} be defined as the node set $S_{uvw} = \{x \in V : \angle_{uvx} \leq \angle_{uvw}\}$. For any node set $V' \subset V$, we define (see Fig. 1)

$$\theta_v(V') = \begin{cases} \theta_{\min} & \text{if } |V'| < 2 \\ \max\{\theta_{\min}, \min_{u,w \in V'}\{\angle_{uvw} : V' \subseteq S_{uvw}\}\} & \text{otherwise .} \end{cases} \quad (1)$$

The beam width $\theta_v(T)$ is hence given as $\theta_v(\Gamma_v(T))$. In Fig. 1, $t_v(T) \in \Gamma_v(T)$ and $t'_v(T) \in \Gamma_v(T)$ are the nodes for which the minimum in (1) is attained in the case $V' = \Gamma_v(T)$ and $|V'| \geq 2$.

The directional minimum energy broadcast problem can then be formulated as

[**D-MEBP**] Find an s-arborescence T such that $p_T = \sum_{v \in V} p_v(T)\theta_v(T)$ is minimized.

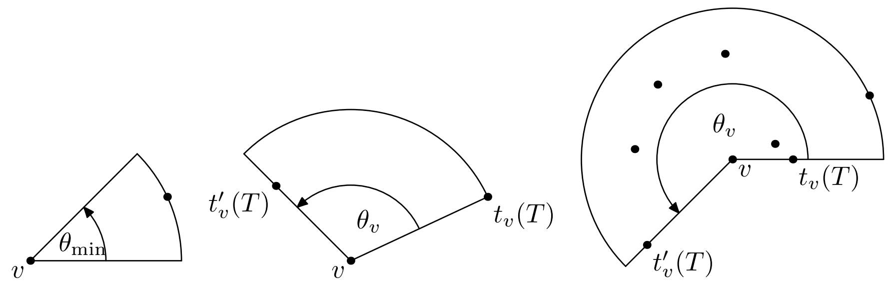

Fig. 1. Examples of beam width $\theta_v(T)$

3 The Reduced Beam procedure

By (1) and the RB principle, any s-arborescence constructing heuristic $\mathcal{H}$ for MEBP can be extended to a D-MEBP heuristic RB-$\mathcal{H}$. This derived heuristic consists of the two steps $\mathcal{H}$ and the RB procedure. The latter simply amounts to computing $\theta_v(T)$ for all $v \in V$, which is accomplished by sorting the children of v according to the angular dimension of their polar coordinates with v as center.

By exploiting the fact that every node has at most $|V|$ children, Wieselthier et al. [5] found that the time complexity of sorting all children of all nodes is bounded by $O(|V|^2 \log |V|)$. However, since there are only a total of $|V| - 1$ children in the arborescence, the time complexity of sorting the children of all nodes is bounded by $O(|V| \log |V|)$. Thus we have the following result.

Theorem 1. *RB has $O(|V| \log |V|)$ time complexity.*

4 Directional BIP

The BIP-algorithm [4] for the omnidirectional version of the problem resembles Prim's algorithm for MST. In each iteration, the best arc from some connected node v to some disconnected node is selected. The algorithms are distinguished in that the selection criterion in BIP is not to minimize arc cost but rather *incremental* arc cost, that is, arc cost minus the cost of the most expensive arc leaving v selected so far.

In [5], the authors adapt BIP to the directional problem. The resulting algorithm is referred to as the Directional BIP (D-BIP) heuristic, which differs from BIP by taking antenna directions and beam widths into account when selecting the next node to be added to the arborescence. In the implementation of D-BIP suggested in [8], the children of every node are maintained as sorted lists. The authors prove that the time complexity of such an implementation is bounded above by $O(|V|^3 \log |V|)$. Through computational experiments, it is also demonstrated that D-BIP outperforms RB-BIP for a large number of test instances.

In the following, we present an implementation of D-BIP that has $O(|V|^2)$ time complexity. It is presented as an extension of BIP, which in its turn can be seen as an extension of Prim's algorithm for MST. Then we demonstrate that these extensions are accomplished without distortion of the quadratic time complexity known to hold for Prim's algorithm.

4.1 An implementation of BIP with quadratic running time

Consider the $O(|A| + |V| \log |V|)$ implementation of Prim's algorithm shown in Table 1, based on the implementation given in [10]. The excluded nodes $V \setminus V_T$ are stored in a priority queue Q. We denote the key value of node v in Q by $\texttt{key}_Q[v]$. The operation $Q.\texttt{extractMin}()$ and $Q.\texttt{extractMax}()$ remove a node with smallest and largest key value, respectively, and return the removed node to the invoking algorithm. An array `parent` is maintained such that for all $v \in V \setminus V_T$, $\texttt{parent}[v]$ is the best parent node of v in V_T.

In all algorithms to follow, we assume that the graph G is represented by a set of adjacency lists $\{\texttt{Adj}[v] : v \in V\}$, where $\texttt{Adj}[v] = \{u : (v, u) \in A\}$.

Table 1. Prim's Algorithm

Prim$(G = (V, A), s, c)$
1 $T = (V_T, A_T) \leftarrow (\{s\}, \emptyset)$
2 *priority queue* $Q \leftarrow V \setminus \{s\}$
3 **for all** $v \in Q$
4     $\texttt{parent}[v] \leftarrow s$
5     $\texttt{key}_Q[v] \leftarrow c_{sv}$
6 **while** $Q \neq \emptyset$
7     $w \leftarrow Q.\texttt{extractMin}()$
8     $v \leftarrow \texttt{parent}[w]$
9     $V_T \leftarrow V_T \cup \{w\}$
10     $A_T \leftarrow A_T \cup \{(v, w)\}$
11   **for all** $u \in \texttt{Adj}[w]$
12   **if** $u \in Q \wedge c_{wu} < \texttt{key}_Q[u]$
13             $\texttt{key}_Q[u] \leftarrow c_{wu}$
14             $\texttt{parent}[u] \leftarrow w$
15 **return** T

Assume the steps in Table 2 are inserted after the for-loop occupying Steps 11-14 in Table 1. In [9], it is proved that this extension results in an implementation of BIP with running time $O(|A| + |V| \log |V|)$.

Table 2. Additional steps needed to extend Prim's algorithm to BIP

for all $u \in$ `Adj[v]`
    if $u \in Q \wedge c_{vu} - c_{vw} < \texttt{key}_Q[u]$
        $\texttt{key}_Q[u] \leftarrow c_{vu} - c_{vw}$
        $\texttt{parent}[u] \leftarrow v$

4.2 Directional BIP as an extension of Prim's Algorithm

Consider a tree $T = (V_T, A_T)$ where $V_T \subset V$, and a node $v \in V_T$ for which $\Gamma_v(T) \neq \emptyset$. Since no three nodes in $\Gamma_v(T)$ are collinear, there exists for each node $u \in \Gamma_v(T)$ a unique $u' \in \Gamma_v(T)$ such that $S_{uvu'} \cap \Gamma_v(T) = \{u, u'\}$. With reference to a polar coordinate system centered at v, u' is the successor of u when sorting $\Gamma_v(T)$ by increasing value of the angular dimension (defined cyclically such that u' is the first node if u is the last). If $|\Gamma_v(T)| = 1$, then $u' = u$.

Define the family of sectors hence induced by node v as $\mathcal{S}_v(T) = \{S_{uvu'} : u \in \Gamma_v(T)\}$. In the example shown in Fig. 2, the sectors induced by v are S_{wvx}, S_{xvy}, S_{yvz} and S_{zvw}.

The figure also illustrates the general fact that $\theta_v(T) = \max\{\theta_{\min}, 2\pi - \max\{\angle_{uvu'} : u \in \Gamma_v(T)\}\}$, which means that if $\theta_v(T) > \theta_{\min}$, then the complementary angle of $\theta_v(T)$ is the angle of a widest sector in $\mathcal{S}_v(T)$. This observation is used to maintain information on the incremental cost of adding a new arc to T.

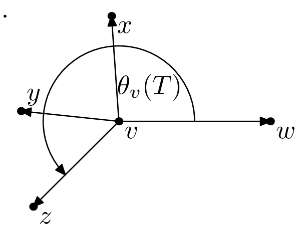

Fig. 2. Sectors induced by v and $\Gamma_v(T)$

For all $u \in V \setminus V_T$ such that $(v, u) \in A$, we need to know the new value of $\theta_v(T)$ given that (v, u) is added to A_T. Consider the case where $|\Gamma_v(T)| > 1$, and let $S_{zvz'}$ and $S_{yvy'}$ be the two widest sectors in $\mathcal{S}_v(T)$ (ties broken arbitrarily), where $\angle_{zvz'} \geq \angle_{yvy'}$. When evaluating the inclusion of u in V_T, we have to take into account how u relates to $S_{zvz'}$ and $S_{yvy'}$:

- If $u \notin S_{zvz'}$, then $S_{zvz'}$ will remain the widest sector in $\mathcal{S}_v(T)$ if (v, u) is added to A_T, and thus $\theta_v(T)$ is unchanged.
- If $u \in S_{zvz'}$, then adding (v, u) to A_T implies that $S_{zvz'}$ leaves $\mathcal{S}_v(T)$, whereas S_{zvu} and $S_{uvz'}$ enter. Consequently, the widest sector in the updated family $\mathcal{S}_v(T)$ is S_{zvu}, $S_{uvz'}$ or $S_{yvy'}$. The new value of $\theta_v(T)$ thus becomes $\max\{\theta_{\min}, 2\pi - \max\{\angle_{zvu}, \angle_{uvz'}, \angle_{yvy'}\}\}$.

It follows that access to the two widest sectors in $\mathcal{S}_v(T)$ is crucial for rapid computation of the incremental cost of adding a potential new arc to A_T. In our implementation of D-BIP, we therefore represent $\mathcal{S}_v(T)$ by a priority queue $\mathcal{S}_v$

Table 3. Additional steps needed to extend Prim's algorithm to D-BIP

for all $u \in \texttt{Adj}[v] \cap Q$
 if $c_{wu}\theta_{\min} < \texttt{key}_Q[u]$
  $\texttt{key}_Q[u] \leftarrow c_{wu}\theta_{\min}$
  $\texttt{parent}[u] \leftarrow w$
if $\Gamma_v(T) = \{w\}$
 priority queue $\mathcal{S}_v \leftarrow \big\{(w,w)\big\}$
 $\texttt{key}_{\mathcal{S}_v}[(w,w)] \leftarrow \angle_{wvw}$
 for all $u \in Q \cap \texttt{Adj}[v]$
  $\theta_{vu} \leftarrow \max\{\theta_{\min}, \min\{\angle_{uvw}, \angle_{wvu}\}\}$
else
 find $(x,x') \in \mathcal{S}_v : w \in S_{xvx'}$
 $\mathcal{S}_v.\texttt{delete}\big((x,x')\big)$
 $\texttt{key}_{\mathcal{S}_v}[(x,w)] \leftarrow \angle_{xvw}$, $\mathcal{S}_v.\texttt{insert}\big((x,w)\big)$
 $\texttt{key}_{\mathcal{S}_v}[(w,x')] \leftarrow \angle_{wvx'}$, $\mathcal{S}_v.\texttt{insert}\big((w,x')\big)$
 $(z,z') \leftarrow \mathcal{S}_v.\texttt{extractMax()}$, $(y,y') \leftarrow \mathcal{S}_v.\texttt{extractMax()}$
 $\mathcal{S}_v.\texttt{insert}\big((y,y')\big)$, $\mathcal{S}_v.\texttt{insert}\big(z,z')\big)$
 for all $u \in Q \cap \texttt{Adj}[v]$
  if $u \in S_{zvz'}$
   $\theta_{vu} \leftarrow \max\left\{\theta_{\min}, 2\pi - \max\left\{\angle_{zvu}, \angle_{uvz'}, \angle_{yvy'}\right\}\right\}$
  else
   $\theta_{vu} \leftarrow \max\{\theta_{\min}, \angle_{zvz'}\}$
for all $u \in \texttt{Adj}[v] \cap Q$
 $\texttt{incCost} \leftarrow \max\{p_v(T), c_{vu}\}\theta_{vu} - p_v(T)\theta_v(T)$
 if $\texttt{incCost} < \texttt{key}_Q[u]$
  $\texttt{key}_Q[u] \leftarrow \texttt{incCost}$
  $\texttt{parent}[u] \leftarrow v$

where $S_{uvu'}$ has key value $\texttt{key}_{\mathcal{S}_v}[(u,u')] = \angle_{uvu'}$. The methods $\mathcal{S}_v.\texttt{insert}\big((u,u')\big)$ and $\mathcal{S}_v.\texttt{delete}\big((u,u')\big)$ are used to add/remove sector $S_{uvu'}$ to/from the queue.

Table 3 shows the steps that replace Steps 11-14 in Table 1 when extending Prim's algorithm to D-BIP. We make use of a matrix $\theta \in \mathbb{R}^A$, where θ_{vu} is the value of $\theta_v(T)$ resulting from the possible inclusion of arc (v,u) in A_T.

To complete the extension of Prim's algorithm to D-BIP, Step 5 in Table 1 has to be changed to

5 $\quad \texttt{key}_Q[v] \leftarrow c_{sv}\theta_{\min}$,

in order to reflect that the cost of adding arc (s,v) is $c_{sv}\theta_{\min}$ rather than c_{sv}. Accordingly, Steps 1-4 in Table 3 are updates of Steps 11-14 in Table 1, taking the minimum beam width into account.

Steps 5-21 concern the updates of $\mathcal{S}_v$ and θ_{vu} after insertion of (v,u) in A_T. Steps 22-26 correspond to the extension made for BIP (Table 2), except that power has been replaced by power times beam width.

Theorem 2. *D-BIP has $O(|V|^2)$ time complexity.*

Proof. All the steps in Table 3 are included in the while-loop in Table 1, which generates $|V|$ iterations. We therefore need to show that each of these steps has at most $O(|V|)$ time complexity.

For the for-loop 1-4, this follows from the analysis of Prim's algorithm.

Given that the priority queues are implemented as Fibonacci heaps, the insertion and key update operations run in constant amortized time, and the operations `delete` and `extractMax` run in $O(\log n)$ amortized time, where n is the maximum number of elements in the queue. Furthermore, any angle $\angle_{uvw}$ is computed in constant time, and checking whether $u \in S_{zvz'}$ is also done in constant time. Thus, each step within the for-loops 8-9, 17-21 and 22-26 runs in constant (amortized) time, and the loops generate at most $|V|$ iterations each. Furthermore, Steps 6-7 and 12-16 have constant and $O(\log|V|)$ time complexity, respectively.

The proof is complete by observing that Step 11 has time complexity $O(|V|)$ since $|\mathcal{S}_v| \leq |V|$. □

5 Conclusions

We have studied how to extend construction heuristics designed for the Minimum Energy Broadcast Problem to the directional version of the problem. Two approaches from the literature, RB and D, were chosen, and we have given fast implementations of both. By virtue of the implementations and analysis given in the current work, the time complexities of previously suggested methods like RB-BIP and D-BIP are reduced by one order of magnitude.

This achievement can be generalized in several directions. Due to the generality of RB and D, our results can be transferred also to other existing and future construction heuristics for MEBP. To simplify the presentation, we have chosen to present the implementations for broadcast routing, but they can easily be adapted to the more general multicast case.

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