Faster Radio Broadcasting in Planar Graphs

Fredrik Manne[1], Shuang Wang, Qin Xin[1]

Department of Informatics
The University of Bergen
P.B. 7800, N-5020, Bergen, Norway
Email: {fredrikm,shuang.wang,xin}@ii.uib.no

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***Abstract*— We study the communication primitive of broadcasting (one-to-all communication) in known topology radio networks, i.e., where for each primitive the schedule of transmissions is precomputed based on full knowledge about the size and the topology of the network. We show that radio broadcasting can be completed in $D + O(\log n)$ time units in planar graphs of size n, diameter D, which improves the currently best known $D + O(\log^3 n)$ time schedule proposed by Elkin and Kortsarz in [6] [SODA'05], and $3D$ time schedule due to Gąsieniec, Peleg and Xin in [11] [PODC'05].**

Keywords: Centralized radio networks, broadcasting, gossiping, planar graphs.

I. Introduction

We consider the following model of a radio network: an undirected connected graph $G = (V, E)$, where V represents the set of nodes of the network and E contains unordered pairs of distinct nodes, such that $(v, w) \in E$ iff the transmissions of node v can directly reach node w and vice versa (the reachability of transmissions is assumed to be a symmetric relation). In this case, we say that the nodes v and w are *neighbours* in G. Note that in a radio network, a message transmitted by a node is always sent to all of its neighbors.

The *degree* of a node w is the number of its neighbours. We use Δ to denote the *maximum degree* of the network, i.e., the maximum degree of any node in the network. The *size of the network* is the number of nodes $n = |V|$.

Communication in the network is synchronous and consists of a sequence of communication steps. In each step, a node v either transmits or listens. If v transmits, then the transmitted message reaches each of its neighbours by the end of this step. However, a node w adjacent to v successfully receives this message iff in this step w is listening and v is the only transmitting node among w's neighbors. If node w is adjacent to a transmitting node but it is not listening, or it is adjacent to more than one transmitting node, then a *collision* occurs and w does not retrieve any message in this step.

The two classical problems of information dissemination in computer networks are the *broadcasting* problem and the *gossiping* problem. The broadcasting problem requires distributing a particular message from a distinguished *source* node to all other nodes in the network. In the gossiping problem, each node v in the network initially holds a message m_v, and the aim is to distribute all messages to all nodes. For both problems, one generally considers as the efficiency criterion the minimization of the time needed to complete the task.

In the model considered here, the running time of a communication schedule is determined by the number of time steps required to complete the communication task. This means that we do not account for any internal computation within individual nodes. Moreover, no limit is placed on the length of a message which one node can transmit in one step. In particular, this assumption plays an important role in the case of the gossiping problem, where it is then assumed that in each step when a node transmits, it transmits all the messages it has collected by that time. (i.e., the ones received and its own one.)

Our schemes rely on the assumption that the communication algorithm can use complete information about the network topology. Such topology-based communication algorithms are useful whenever the underlying radio network has a fairly stable topology/infrastructure. As long as no changes occur in the network topology during the execution of the algorithm, the tasks of broadcasting and gossiping will be completed successfully. In this extended abstract we do not touch upon reliability issues. However, we remark that it is possible to increase the level of fault-tolerance in our algorithms, at the expense of some small extra time consumption. We defer this issue to the extended version of this paper.

Our results. We provide a new (efficiently computable) deterministic schedule that uses $D + O(\log n)$ time units to complete the broadcasting task in any planar graph of size n, diameter D. This significantly improves on the previously known best schedule, i.e., the $D + O(\log^3 n)$ schedule of [6]. Remarkably, our new broadcasting scheme also improves the $3D$-time schedule in [11] for large diameter D.

Related work. The work on communication in known topology radio networks was initiated in the context of the broadcasting problem. In [3], Chlamtac and Weinstein prove that the broadcasting task can be completed in time $O(D \log^2 n)$ for every n-vertex radio network of diameter D. An $\Omega(\log^2 n)$ time lower bound was proved for the family of graphs of radius 2 by Alon *et al* [1]. In [6], Elkin and Kortsarz give an efficient deterministic construction of a broadcasting schedule of length $D + O(\log^4 n)$ together with a $D + O(\log^3 n)$ schedule for planar graphs. Recently, Gąsieniec, Peleg and Xin [11] showed that a $D + O(\log^3 n)$ schedule exists for the broadcast task, that works in *any* radio network. In the same paper, the authors also provide an optimal randomized broadcasting schedule of length $D + O(\log^2 n)$ and a new broadcasting schedule using fewer than $3D$ time slots on planar graphs. Very recently, a $24D + O(\log^2 n)$ time deterministic broadcasting schedule for any radio network was proposed by Kowalski and Pelc in [13]. This is asymptotically optimal unless $NP \subseteq BPTIME(n^{\mathcal{O}(\log \log n)})$ [13]. Nonetheless, for large D, a $D + O(\frac{\log^3 n}{\log \log n})$ time broadcasting scheme outperforms the one in [13], which was proposed by Cicalese, Manne and Xin in [4] very recently. Efficient radio broadcasting algorithms for several special types of network topologies can be found in Diks *et al.* [5]. For general networks, however, it is known that the computation of an optimal (radio) broadcast schedule is NP-hard, even if the underlying graph is embedded in the plane [2], [15].

Radio gossiping in networks with known topology was first studied in the context of radio communication with messages of limited size, by Gąsieniec and Potapov in [9]. They also proposed several optimal or close to optimal $O(n)$-time gossiping procedures for various standard network topologies, including lines, rings, stars and free trees. For general topology radio network a $O(n \log^2 n)$ gossiping scheme is provided and it is proved that there exists a radio network topology in which the gossiping (with unit size messages) requires $\Omega(n \log n)$ time. In [14], Manne and Xin show the optimality of this bound by providing an $O(n \log n)$-time gossiping schedule with unit size messages in any radio network. The first work on radio gossiping in known topology networks with arbitrarily large messages is [10], where several optimal gossiping schedules are shown for a wide range of radio network topologies. For arbitrary topology radio networks, an $O(D + \Delta \log n)$ schedule was given by Gąsieniec, Peleg and Xin in [11]. Very recently, Cicalese, Manne and Xin [4] provided a new (efficiently computable) deterministic schedule that uses $O(D + \frac{\Delta \log n}{\log \Delta - \log \log n})$ time units to complete the gossiping task in any radio network of maximum degree $\Delta = \Omega(\log n)$.

II. Broadcasting in Planar Graphs with Known Topology

In this section we present the idea of a deterministic algorithm that generates a schedule for completing the broadcasting task in a planar graph in time $D + O(\log n)$. Our schedule is based on the notion of a *gathering spanning tree* as given in [11].

A. Preliminaries

We first recall the following recursive ranking procedure of nodes in a tree (see [11]). Leaves have rank 1. Next consider a node v and the set Q of its children and let r_{max} be the maximum rank of the nodes in Q. If there is a unique node in Q of rank r_{max} then set the rank of v to r_{max}, otherwise set the rank of v to $r_{max} + 1$.

Lemma 1: The largest rank in a tree of size n is bounded by $\lceil \log n \rceil$. (see [11]).

Given any graph G with central node c, the nodes are partitioned into consecutive layers $L_i = \{v \mid dist(c, v) = i\}$, for $i = 0, .., r$ where r is the radius of G. A *gathering spanning tree GST* of G is a BFS spanning tree T of G rooted at c, such that T is ranked as above and that also satisfies the following condition: every node in L_{j+1} of rank i is at most adjacent to one node in L_j also of rank i, and thus if all the nodes of rank i in L_j transmit at the same time then the messages will be received by the nodes in L_{j+1} of rank i successfully without any collision.

The following lemma was shown in [11].

Lemma 2: There exists a polynomial time construction of a GST in any graph G.

Figure 1 shows how a gathering spanning tree can be constructed from a graph G.

In the following we assume a ranked *GST* in G given by the parent relation on each node. For clarity of presentation, we use the same definitions as in [11].

Definition 3: the *rank sets*: $R_i = \{v \mid \texttt{rank}(v) = i\}$, where $1 \le i \le r_{max} \le \lceil \log n \rceil$.

Definition 4: the *fast transmission set*: $F_i^k = \{v \mid v \in L_k \cap R_i \text{ and } parent(v) \in R_i\}$. We also define $F_i = \bigcup_{k=1}^{D} F_i^k$ and $F = \bigcup_{i=1}^{r_{max}} F_i$.

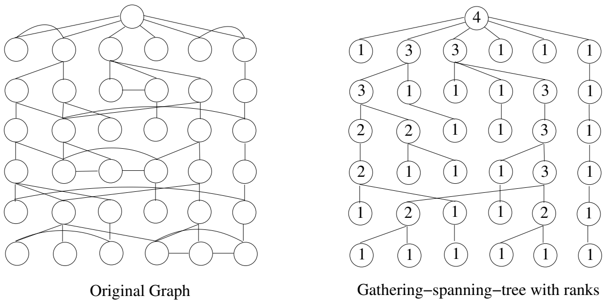

Fig. 1. Creating a gathering spanning tree.

Definition 5: the *slow transmission set*: $S_i^k = \{v \mid v \in L_k \cap R_i \text{ and } parent(v) \in R_j, j > i\}$. We also define $S_i = \bigcup_{k=1}^{D} S_i^k$ and $S = \bigcup_{i=1}^{r_{max}} S_i$.

Lemma 6: In a planar graph, all nodes in set S_{i+1} of a GST can be informed by their parents in L_i of the GST in three time units.

Proof. In [11], it states that all nodes in one partition can be informed by another partition in a bipartite planar graph (in this case two consecutive BFS layers) in three time units. The solution is based on the use of the *3-step subschedule Procedure PB*. (See an example in Figure 2.) Note that during this process the source message m_s received by a node $v \in S_{i+1}$ may be delivered by some other nodes in L_i of GST rather than exactly by its parent in the GST.

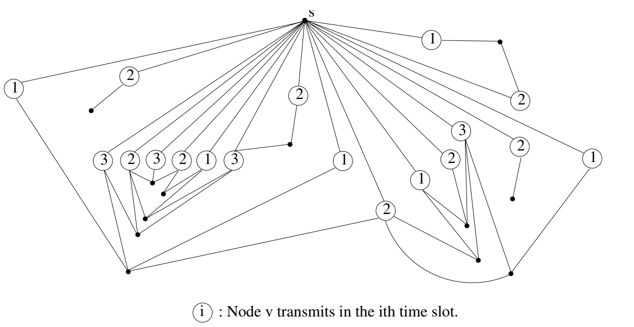

Fig. 2. An example of Procedure PB. (All solid black nodes in L_2 will receive the source message from the white hollow nodes in L_1 after three time slots. Note all white hollow nodes had already received the source message from s in the previous time slot.)

B. Deterministic construction of a $D + O(\log n)$ broadcasting schedule

The deterministic algorithm uses the ranked gathering spanning tree GST, on this occasion rooted in the *source node* s. The algorithm uses *fast* and *slow* transmissions that partition the set of nodes to the sets F and S, where the broadcast message is disseminated from the root s (using parent-child connections) towards the leaves of the tree.

Let us start with an overview of the broadcast process from the point of view of a copy of the message that was eventually received at some leaf a of the tree. Note that this message does not necessarily have to follow the unique shortest path $p(a)$ leading from the root of the tree to a. In fact, there are many paths on which the message could be forwarded, some of which do not even need to be shortest paths. For the sake of the time complexity analysis, however, we fix our attention on the path $p(a)$ and argue about the potential progress of the message along this path.

Conceptually, the path $p(a)$ is broken down into segments

$$p(a) = \langle p_1^F(a), p_1^S(a), p_2^F(a), p_2^S(a), \ldots, p_q^F(a), p_q^S(a) \rangle ,$$

where each $p_i^F(a)$ is a segment consisting of fast transmission edges (i.e., edges leading from $parent(v)$ to v, both of the same rank) and each $p_i^S(a)$ is a single edge (u, w) where u is a node on layer L_k for some k, w is a node on layer L_{k+1} and $\mathtt{rank}(u) > \mathtt{rank}(w)$. We refer to such edges (u, w) as slow transmission edges (see Figure 3 for an example). (Note that some of the segments $p_i^F(a)$ may be empty.)

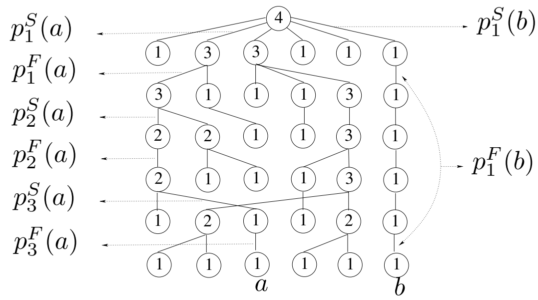

Fig. 3. An example of the path partition.

Again, we stress that in reality, the message need not follow this path. Nevertheless, we may consider the "progress" of the message along this path, by measuring the delay from the time the message is already available at some node v on the path $p(a)$ to the time the message has already reached the following node w on the path (though not necessarily via a transmission from v). Hence conceptually, the message progress can be viewed as traversing the path $p(a)$ by alternating (flipping) between chains $p_i^F(a)$ of fast transmission edges connecting nodes of the same rank and slow transmission steps over edges $p_i^S(a)$, connecting high rank nodes to lower rank nodes.

Next we describe the schedule governing these transmissions. During the broadcasting process the nodes in the tree use the following pattern of transmissions. Let $r_{max} \leq \lceil \log n \rceil$ be the largest rank in the tree. Consider a node v of rank $1 \leq j \leq r_{max}$ on BFS layer L_i with a child w also of rank j at the next BFS layer L_{i+1}. Then v is set to perform a fast transmission to w in time steps t satisfying $t \equiv i+6(r_{max}-j) \bmod 6r_{max}$. Observe that in real terms, v will perform such a fast transmission exactly once, on the first appropriate time slot after it receives the message for the first time. The slow transmissions at the BFS layer L_i are performed in the time steps t satisfying $t \equiv i+3 \bmod 6$. Note that this pattern of transmissions separates the fast and the slow transmissions at any BFS layer by three units of time. Thus there are no collisions between the fast and the slow transmissions at the same BFS layer. The pattern also ensures that at any time step, transmissions are performed on BFS layers at distances that are multiples of 3 apart. Thus there will be no conflicts between transmissions coming from different BFS layers.

Note also that once the broadcast message arrives at the first node v of a fast segment $p_i^F(a)$ of the route with a particular rank j, it may have to wait for at most 6 (e.g. $O(1)$) time steps if the parent of v in GST has rank $j+1$, otherwise at most $6(rank(parent(v)) - j)$, but then, when finally transmitted to the next BFS layer, it will be forwarded through the fast segment $p_i^F(a)$ without further delays.

Once reaching the end node u of the fast segment $p_i^F(a)$, the message has to be transmitted from some node on u's BFS layer to the next node w on $p(a)$, which is of lower rank, using a slow transmissions mechanism. For slow transmissions, the algorithm uses the $O(1)$ transmission due to Lemma 6, which is based on procedure PB proposed by Gąsieniec, Peleg and Xin in [11]. The slow transmission mechanism is run repeatedly in a periodic manner at every BFS layer of the tree. In particular, at any BFS layer, the steps of the slow transmission procedure PB are performed in every 6th step of the broadcasting schedule.

Hence, suppose the broadcast message traversing towards any destination a in the tree has reached a node u of BFS layer L_j on its path $p(a)$, such that the next edge (u, w) on the path is a slow transmission edge. It is possible that neither u nor any other neighbor of w on BFS layer L_j participates in the current activation of procedure PB on L_j (possibly because neither of those nodes had the message at the last time the procedure was activated). Nevertheless, u will participate in the next activation of procedure PB on BFS layer L_j, which will be started within at most $O(1)$ time (namely, the time required for the current activation to terminate). Moreover, it is guaranteed that by the time that activation of procedure PB terminates, w will have the message (although it may get it from any of its neighbors in L_j, and not necessarily directly from u). Hence this entire stage can be thought of as a slow transmission operation on the edge (u, w), taking a total of at most $O(1)$ time steps due to Lemma 6.

In the view of these observations, the total time required for the broadcast message to reach a leaf a in the tree can be bounded as follows. Let D_i, for $1 \leq i \leq r_{max}$, denote the length of $p^F(a)$, the ith fast segment of the route $p(a)$ used by the broadcast message that has reached to a. Thus the time required to communicate a is bounded by $O(1)+D_1+O(1)+D_2+\ldots+O(1)+D_{r_{max}} \leq D+O(\log n)$ for the fast transmissions plus $r_{max} \cdot O(1) = O(\log n)$ for the slow transmissions, yielding a total of $D+O(\log n)$. Thus we have the following theorem.

Theorem 7: There exists a deterministic polynomial time algorithm that constructs, for any planar graph of size n and diameter D, a broadcasting schedule of length $D+O(\log n)$.

C. An example of the broadcasting schedule

In this section, we give an example for our broadcasting schedule based on a notion of the *gathering spanning tree*.

(i) Construction of a gathering spanning tree. (See Figure 4.)

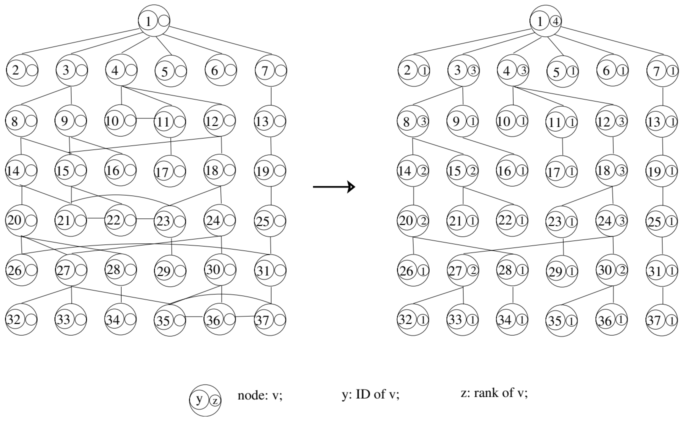

Fig. 4. Construct a gathering spanning tree from the original planar graph.

(ii) The transmission pattern.

Steps	Transmissions
(1)	$1 \rightarrow 2 \mid\mid 1 \rightarrow 3 \mid\mid 1 \rightarrow 4 \mid\mid$
	$1 \rightarrow 5 \mid\mid 1 \rightarrow 6 \mid\mid 1 \rightarrow 7 \mid\mid$
(2)	$3 \rightarrow 8 \mid\mid 4 \rightarrow 12 \mid\mid$

(3)	$12 \to 18 \mid\mid$
(4)	$18 \to 24 \mid\mid 3 \to 9 \mid\mid 4 \to 10 \mid\mid 4 \to 11 \mid\mid$
(5)	$8 \to 14 \mid\mid 8 \to 15 \mid\mid$
(6)	$15 \to 21 \mid\mid 15 \to 22 \mid\mid 18 \to 23 \mid\mid$
(7)	$24 \to 27 \mid\mid 24 \to 30 \mid\mid$
(8)	$27 \to 32 \mid\mid 27 \to 33 \mid\mid 30 \to 36 \mid\mid$
(9)	$14 \to 20 \mid\mid$
(10)	$20 \to 26 \mid\mid 20 \to 28 \mid\mid 7 \to 13 \mid\mid$
(11)	$9 \to 16 \mid\mid 11 \to 17 \mid\mid 13 \to 19 \mid\mid$
	$30 \to 35 \mid\mid$
(12)	$19 \to 25 \mid\mid$
(13)	$23 \to 29 \mid\mid 25 \to 31 \mid\mid$
(14)	$28 \to 34 \mid\mid 31 \to 37 \mid\mid$

III. Conclusion

We have proposed an efficient (polynomial time) construction of a deterministic schedule that performs the radio broadcasting in time $D + O(\log n)$ in planar graphs of size n, diameter D. The evident open problem is whether there exists a deterministic broadcasting schedule of time $D + O(1)$ for planar graphs.

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