Canonical tree-decompositions and nested separation systems.

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Abstract

We provide a method to construct tree-decompositions of a graph $G$ which separate their ‘highly connected parts’. They are canonical in the sense that their construction only depends on properties of $G$ that stay invariant under automorphisms of $G$.

Given a set of separations $S$ and a set $I$ of ‘objects’ distinguishable by $S$ we construct a nested subsystem $\mathcal{N} \subseteq S$ which still distinguishes all elements of $I$—provided the pair $(S, I)$ meets certain conditions. This method is very flexible and adapts to different notions of ‘highly connected parts’. In that way we are able to establish major improvements on previous results by Robertson and Seymour and by Dunwoody and Krön: We find canonical tree-decompositions which distinguish all the maximal tangles and all the maximal $k$-inseparable sets, for all $k$ simultaneously.