Canonical tree-decompositions and nested separation systems.

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Abstract

We provide a method to construct tree-decompositions of a graph G which separate their '*highly connected parts*'. They are canonical in the sense that their construction only depends on properties of G that stay invariant under automorphisms of G.

Given a set of separations $\mathcal{S}$ and a set $\mathcal{I}$ of 'objects' distinguishable by $\mathcal{S}$ we construct a nested subsystem $\mathcal{N} \subseteq \mathcal{S}$ which still distinguishes all elements of $\mathcal{I}$—provided the pair $(\mathcal{S}, \mathcal{I})$ meets certain conditions. This method is very flexible and adapts to different notions of '*highly connected parts*'. In that way we are able to establish major improvements on previous results by Robertson and Seymour and by Dunwoody and Krön: We find canonical tree-decompositions which distinguish all the maximal tangles and all the maximal k-inseparable sets, for all k simultaneously.