Subexponential Algorithms for Partial Cover Problems

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Abstract

Partial Cover problems are optimization versions of fundamental and well studied problems like Vertex Cover and Dominating Set. Here one is interested in covering (or dominating) the maximum number of edges (or vertices) using a given number (k) of vertices, rather than covering all edges (or vertices). In general graphs, these problems are hard for parameterized complexity classes when parameterized by k. It was recently shown by Amini et. al. [*FSTTCS 08*] that Partial Vertex Cover and Partial Dominating Set are fixed parameter tractable on large classes of sparse graphs, namely H-minor free graphs, which include planar graphs and graphs of bounded genus. In particular, it was shown that on planar graphs both problems can be solved in time $2^{\mathcal{O}(k)}n^{\mathcal{O}(1)}$.

During the last decade there has been an extensive study on parameterized subexponential algorithms. In particular, it was shown that the classical Vertex Cover and Dominating Set problems can be solved in subexponential time on H-minor free graphs. The techniques developed to obtain subexponential algorithms for classical problems do not apply to partial cover problems. It was left as an open problem by Amini et al. [*FSTTCS 08*] whether there is a subexponential algorithm for Partial Vertex Cover and Partial Dominating Set. In this paper, we answer the question affirmatively by solving both problems in time $2^{\mathcal{O}(\sqrt{k})}n^{\mathcal{O}(1)}$ not only on planar graphs but also on much larger classes of graphs, namely, apex-minor free graphs. Compared to previously known algorithms for these problems our algorithms are significantly faster and simpler.

1 Introduction and Motivation

A generic instance of a covering problem consists of a family of sets over an universe and the objective is to cover the universe with as few sets from the family as possible. Covering problems are basic problems not only in combinatorial optimization and algorithms but occur naturally in variety of applications. One of the prominent covering problems is the classical Set Cover problem. Other classical problems in the framework of covering include well known problems like Vertex Cover, Dominating Set, Facility Location, k-Median, k-Center problems, on which hundreds of papers have been written.

As the name suggests, in partial cover problems one is interested in covering as much of the universe, if not the entire universe. This makes the partial cover problems natural generalizations of the well known covering problems. More precisely, in the partial covering problem, for a given integer $t \geq 0$, we want to cover at least t elements using as few objects (vertices or edges) as possible. For an example, in Partial Vertex Cover (PVC), the goal is to cover at least t edges with the minimum number of vertices while in Partial Dominating Set (PDS) the goal is to dominate at least t vertices of the input graph with the minimum number of vertices.

Partial cover problems have been investigated extensively and are well understood in the context of polynomial time approximation [2, 4, 3, 5, 16, 18] and parameterized complexity [1, 4, 24, 25, 23, 27]. In this paper we study partial cover problems defined on graphs

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namely Partial Vertex Cover and Partial r-Dominating Set from the view point of parameterized algorithms. Partial Vertex Cover is defined as follows.

> Partial Vertex Cover (PVC): Given a graph $G = (V, E)$ and positive integers k and t, check whether there exists a set of vertices $C \subseteq V$ such that $|C| \leq k$ and there are at least t edges incident to C.

The Partial r-Dominating Set is a generalization of Dominating Set and is defined as follows.

> Partial r-Dominating Set (P-r-DS): Given a graph $G = (V, E)$ and positive integers k, r and t, determine whether there exists a set of vertices $D \subseteq V$ such that $|D| \leq k$ and there are at least t vertices at distance at most r from some vertex in D.

In parameterized algorithms, for decision problems with input size n, and a parameter k, the goal is to design an algorithm with runtime $f(k) \cdot n^{O(1)}$, where f is a function of k alone. Problems having such an algorithm are said to be fixed parameter tractable (FPT). There is also a theory of hardness using which one can identify parameterized problems that are not amenable to such algorithms. This hardness hierarchy is represented by $W[i]$ hierarchy for $i \geq 1$. For an introduction and more recent developments see the books [13, 14, 29]. In this paper, we always parameterize a problem by the size of the cover, that is, the positive integer k.

Most of the research on partial cover problems in parameterized complexity has considered the number of objects to be covered (t) as a parameter rather than the the size of the cover (k). Bläser [4] initiated the study of partial cover problems parameterized by t and obtained a randomized algorithm with running time $5.45^t n^{\mathcal{O}(1)}$ for PDS. Kneis et al. [25] improved this algorithm and obtained a randomized algorithm with running time $(4+\epsilon)^t n^{\mathcal{O}(1)}$ for every fixed $\epsilon > 0$. Recently, Koutis and Williams [27] obtained an even faster randomized algorithm for PDS, which runs in time $2^t n^{\mathcal{O}(1)}$. Kneis et al. [24] studied the PVC problem when parameterized by the number edged to be covered (t) and obtained a randomized algorithm running in time $2.0911^t n^{\mathcal{O}(1)}$. The algorithm for PVC was recently improved by Kneis et al. [23]. They obtain a randomized algorithm with running time $1.2993^t n^{\mathcal{O}(1)}$ and a deterministic algorithm with running time $1.396^t n^{\mathcal{O}(1)}$ for PVC. When parameterized by the size of cover k, PVC is known to be $W[1]$-complete [17]. The P-r-DS problem being a generalization of Dominating Set is also known to be $W[2]$-hard on general graphs when parameterized by the cover size. Amini et al. [1] considered these problems with the size of the cover k being the parameter and initiated a study of these problem on sparse graphs namely planar graphs, apex minor free graphs and H-minor free graphs. They obtained algorithms with running time $2^{\mathcal{O}(k)} n^{\mathcal{O}(1)}$ for PVC and P-r-DS and left an open question of whether these problems have an algorithm with running time $2^{o(k)} n^{\mathcal{O}(1)}$, like their non partial counterpart on planar graphs or more generally on H-minor free graphs. In this paper we answer this question in affirmative and obtain algorithms with running time $2^{\mathcal{O}(\sqrt{k})} n^{\mathcal{O}(1)}$ for PVC and P-r-DS on planar graphs and more general classes of sparse graphs, namely, apex-minor free graphs.

Most of the known sub-exponential time algorithms on planar graphs, graphs of bounded genus, apex minor free graphs and H-minor free graphs are based on the meta-algorithmic theory of bidimensionality, developed by Demaine et al. [7]. The bidimensionality theory is based on algorithmic and combinatorial extensions to various parts of Graph Minors Theory of Robertson and Seymour [30] and provides a simple criteria for checking whether a parameterized problem is solvable in subexponential time on sparse graphs. The theory applies to the graph problems that are *bidimensional* in the sense that the value of the solution for the problem in question on $k \times k$ grid or "grid like graph" is at least $\Omega(k^2)$ and the value of solution decreases while contracting or sometime deleting the edges. Problems that are bidimensional include k-Feedback Vertex Set, k-Edge Dominating Set, k-Leaf Spanning Tree, k-Path, k-rDominating Set, k-Vertex Cover and many others. We refer to surveys by Demaine and Hajiaghayi [10] and Dorn et al. [12] for further details on bidimensionality and subexponential parameterized algorithms. But neither PVC nor P-r-DS are bidimensional problems and hence this theory is *not amenable* to our problems.

Our subexponential time algorithms for PVC and P-r-DS are based on a technique used to solve the classical Disjoint Path problem in the Graph Minors Theory of Robertson and Seymour [31], called *irrelevant vertex* argument. The technique can be described as follows, in polynomial time we find a vertex which is irrelevant for the solution and hence can be deleted and when we can not find an irrelevant vertex, we show that the reduced instance has bounded treewidth. This technique has recently been used to solve several problems around finding disjoint paths [19, 20, 21, 22, 26]. To obtain subexponential time algorithms for PVC and P-r-DS we introduce a notion of "lexicographically smallest" solution and use its properties to obtain an irrelevant vertex in the graph. When we can not find any irrelevant vertex then we are able to show that that the treewidth of the reduced graph is at most $\mathcal{O}(\sqrt{k})$. Once we have a sublinear bound on the treewidth of the input graph, we can solve the problem in $2^{\mathcal{O}(\sqrt{k})}n^{\mathcal{O}(1)}$ time using dynamic programming over graphs of bounded treewidth. Our results are based on a simple but powerful observation relating lexicographically least solutions and r-dominating sets of size at most k.

2 Preliminaries

Let $G = (V, E)$ be an undirected graph where V is the set of vertices and E is the set of edges. We denote the number of vertices by n and number of edges by m. For a subset $V' \subseteq V$, by $G[V']$ we mean the subgraph of G induced by V'. By $N(u)$ we denote (open) neighborhood of u that is set of all vertices adjacent to u and by $N[u] = N(u) \cup \{u\}$. Similarly, for a subset $D \subseteq V$, we define $N[D] = \cup_{v \in D} N[v]$. The *distance* $d_G(u, v)$ between two vertices u and v of G is the length of the shortest path in G from u to v. For a given vertex $v \in V$ by $\partial(v)$ we denote the set of edges which are incident with v. For a subset $X \subseteq V$, $\partial(S) = \cup_{v \in S} \partial(v)$.

Given an edge $e = (u, v)$ of a graph G, the graph G/e is obtained by contracting the edge (u, v) that is we get G/e by identifying the vertices u and v and removing all the loops and duplicate edges. A *minor* of a graph G is a graph H that can be obtained from a subgraph of G by contracting edges. A graph class $\mathcal{C}$ is *minor closed* if any minor of any graph in $\mathcal{C}$ is also an element of $\mathcal{C}$. A minor closed graph class $\mathcal{C}$ is *H-minor-free* or simply *H-free* if $H \notin \mathcal{C}$. A graph H is called an apex graph if the removal of one vertex makes it a planar graph.

A *tree decomposition* of a graph $G = (V, E)$ is a pair (X, T) where T is a tree on vertex set $V(T)$ whose vertices we call *nodes* and $X = (\{X_i \mid i \in V(T)\})$ is a collection of subsets of V such that

1. $\bigcup_{i \in V(T)} X_i = V$,
2. for each edge $(v, w) \in E$, there is an $i \in V(T)$ such that $\{v, w\} \in X_i$, and
3. for each $v \in V$ the set of nodes $\{i \mid v \in X_i\}$ forms a subtree of T.

The *width* of a tree decomposition $(\{X_i \mid i \in V(T)\}, T)$ equals $\max_{i \in V(T)}\{|X_i| - 1\}$. The *treewidth* of a graph G is the minimum width over all tree decompositions of G. We use notation $\mathbf{tw}(G)$ to denote the treewidth of a graph G.

Given a graph $G = (V, E)$ a set of vertices D of V is called a *r-dominating set* for G if $N_r(D) = V$. For $r = 1$ the set D is called a *dominating set*. In the r-Dominating Set problem, we are given a graph $G = (V, E)$ and the objective is to find the smallest sized D such that $N_r(D) = V$.

3 Subexponential algorithm for Partial Vertex Cover

In this section we consider the PVC problem. In fact we will solve a slightly more general problem, that is, given an undirected graph, a non negative integer k, we find the *maximum* number of edges that can be covered by a subset of at most k vertices. The decision version of the problem is precisely PVC. If the maximum number of edges covered by any vertex set of size at most k is at least t then we return "yes" else we return "no".

The key idea of the algorithm is to identify a set of *irrelevant* vertices, I, which can be deleted without destroying at least one set $C \subseteq V$ such that $|C| \leq k$ and $|\partial(C)| \geq t$, if such a set exists. Then we will show that the $\mathbf{tw}(G[V \setminus I]) \leq \mathcal{O}(\sqrt{k})$ and hence the dynamic programming over graphs of bounded treewidth can be applied. To identify a set of irrelevant vertices we introduce the notion of *lexicographically smallest solution*.

Definition 1. *Given a graph $G = (V, E)$ and an ordering $\sigma = v_1 \dots v_n$ of the vertices in V, if X is lexicographically smaller than Y then we denote it by $X \leq_\sigma Y$. We call a set $C \subseteq V$ the* lexicographically smallest solution *for PVC if for any other solution C' for the PVC we have that $C \leq_\sigma C'$.*

Let $\sigma = v_1 v_2 \dots v_n$ be an ordering of the vertices such that the vertices are in non increasing order of their degrees, with ties being broken arbitrarily. That is,

$$d(v_1) \geq d(v_2) \cdots \geq d(v_{n-1}) \geq d(v_n).$$

Throughout this section, we will assume that the vertex set of the input graph is ordered by *this* fixed ordering σ and denote the graph by $G = (V_\sigma, E)$ to *emphasize* the fact that the vertex set is order with respect to σ. By V_σ^i we denote the vertex set $v_1 \dots v_i$. Our goal will be to find the lexicographically smallest solution for PVC. The algorithm is based on the following properties of the lexicographically smallest solution for PVC.

Lemma 1. *Let $G = (V_\sigma, E)$ be an yes instance to* PVC*, $C = \{u_{i_1}, \dots, u_{i_k}\}$ be the lexicographically smallest solution for* PVC *and $u_{i_k} = v_j$ for some j. Then C is a dominating set of size at most k for $G[V_\sigma^j]$.*

Proof. Let us assume to the contrary that C is not a dominating set for $G[V_\sigma^j]$. Then there exists a vertex v_i, $1 \leq i < j$ such that $N[v_i] \cap C = \emptyset$. Set $C' := C \setminus \{v_j\} \cup \{v_i\}$. We claim that C' covers at least as many edges as are covered by C. That is, $|\partial(C')| \geq |\partial(C)|$. Since $d(v_i) \geq d(v_j)$, we have that

$$|\partial(C')| \geq |\partial(C)| - d(v_j) + d(v_i) \geq |\partial(C)|.$$

This is because the edges covered by v_i are not covered by any element of $C - \{v_j\}$. Hence, $|C'| = |C|$, C' is lexicographically smaller than C and $|\partial(C')| \geq |\partial(C)|$ a contradiction to the choice of C. □

We also need the following results for our algorithm.

Lemma 2. *Let G be a n-vertex graph excluding an apex graph H as a minor. If G has a r-dominating set of size at most k, then G has treewidth at most $c_H r\sqrt{k} = \mathcal{O}(r\sqrt{k})$, where c_H is a constant depending only on the size of H.*

Lemma 2 follows from the fact that the size of r-dominating set is a "contraction bidimensional" parameter and that if a contraction bidimensional parameter has value at most k on a graph G which excludes an apex graph H as a minor then $\mathbf{tw}(G) \le \mathcal{O}(r\sqrt{k})$ [6, 8, 15] . We will use the following known algorithm to solve PVC on graphs of bounded treewidth.

Lemma 3 ([28])**.** *Let G be an undirected graph such that the treewidth of G is at most w. Then in time $2^w n^{\mathcal{O}(1)}$ we can find a subset C of at most k vertices that cover the maximum number of edges of G.*

For our proof we also need the following result by Demaine and Hajiaghayi to obtain a polynomial time approximation scheme (PTAS) for r-Dominating Set.

Lemma 4 ([9])**.** *There is a PTAS for r-*Dominating Set *on apex minor free graphs.*

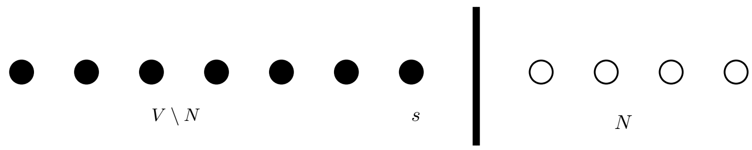

Figure 1: The Algorithmic Schema

The basic schema of the algorithm is as follows. We start with the vertex set V_σ and scan the vertices in the *reverse* order of $\sigma = v_1 v_2 \dots v_n$. That is, we scan the vertices in the order $v_n v_{n-1} \dots v_2 v_1$. The algorithm can be viewed as having a stick, initially positioned to the right of v_n which we *slide* towards its left if the vertex to its left satisfies certain properties. See Figure 1. At any intermediate stage, we have a vertex set N which are the vertices in the original order σ, to the right of the stick. The vertex set s is the first vertex to the left of the stick. The stick represents the fact that the lexicographically smallest solution C we are looking for lies completely in $V \setminus N$, that is, $C \subseteq V \setminus N$. To slide the stick we do as follows. Let $s = v_j$ for some j. Now we check whether $G[V_\sigma^j]$ has a dominating set of size "roughly k". If not, we slide the stick to one position left. Else we find an appropriate induced subgraph $G' = (V', E')$ of G such that $\mathbf{tw}(G') \le \mathcal{O}(\sqrt{k})$ and G has a set C of size at most k such that $|\partial(C)| \ge t$ if and only if there exists a set $C' \subseteq V'$ such that $|C'| \le k$ and $|\partial(C')| \ge t$. A formal description of our algorithm for partial vertex cover is given in Figure 2. The Algo-PC is called with the parameter $(G = (V_\sigma, E), k, \epsilon, \emptyset)$. Now we state our main theorem for this section.

Theorem 1. *Let $G = (V, E)$ a graph that excludes an apex graph H as a minor and k and t be a positive integers. Then in $2^{\mathcal{O}(\sqrt{k})} n^{\mathcal{O}(1)}$ time we can determine whether there exists a subset $C \subseteq V$ of size at most k such that $|\partial(C)| \ge t$.*

Proof. We argue the correctness of the algorithm. In the first part of the algorithm we try to identify the subset N of vertices such that it does not intersect with the lexicographically least solution C we are looking for. We iteratively run through the vertices in the reverse order and try to maintain the invariant that N is a subset of the vertices that does not intersect with the lexicographically least solution. Initially N is empty, so the invariant trivially holds. The set N only grows if in any step, the PTAS algorithm of Lemma 4 finds a dominating set of $G[V \setminus N]$ of size more than $(1+\epsilon)k$. Let v_p be the largest indexed vertex in $V \setminus N$, that is, v_p is to the left of the set N in the ordering σ. Now by Lemma 5, we know that if $v_p \in C$ then $G[V \setminus N]$ has a dominating set of size at most k and hence the PTAS from Lemma 4 would find an approximate dominating set of size at most $(1+\epsilon)k$. This implies that $v_p \notin C$ and hence we can safely place v_p in N. This proves the correctness of the first part.

ALGO-PC($G = (V_\sigma, E), k, \epsilon, N$)
(Here G is a graph with vertices ordered in non increasing order σ of their degrees , k a non negative integer, $\epsilon > 0$ is an arbitrary fixed constant, N is a set of vertices (initially $\emptyset$), and the goal is to find a subset of $V \setminus N$ of size at most k that covers the maximum number of edges of $G = (V, E)$.)

1. Let $p := n$.
2. While there does not exist a dominating set of size at most $(1+\epsilon)k$ for $G[V_\sigma^p]$ (determined using Lemma 4)
   - set $N := N \cup \{v_p\}$ and $p := p - 1$.

   endwhile
3. Let $I = \{u \mid u \in N, N(u) \subseteq N\}$ and set $V' = V \setminus I$. Find a tree-decomposition (U, T) of $G[V']$ using the constant factor approximation algorithm of Demaine et al. [11] for computing the treewidth of H-minor free graph.
4. Apply Lemma 3 to find a subset C' of size at most k of $G[V']$ which covers the maximum number of edges.

Figure 2: Description of the partial cover Algorithm

Note that edges in $G[N]$ will not be covered by C, and hence vertices in N that have neighbors only in N are collected in the set I and deleted at the end. The set I is the irrelevant set of vertices we were looking for. Let $V' = V \setminus I$. Thus we have shown that G has a set C of size at most k such that $|\partial(C)| \geq t$ if and only if there exists a set $C' \subseteq V'$ such that $|C'| \leq k$ and $|\partial(C')| \geq t$. Now applying Lemma 3 we find a subset C' of size at most k of $G[V']$ which covers the maximum number of edges. So if $|\partial(C')| \geq t$ then we return "yes" else we return "no". The correctness of this step follows from Lemma 3.

Now we analyze the time complexity of the algorithm. We know that when the algorithm exits the while loop, $G[V \setminus N]$ has a dominating set of size at most $(1+\epsilon)k$. Let D be a dominating set of $G[V \setminus N]$ of size at most $(1+\epsilon)k$. This implies that D is a 2-*dominating set* of $G[V']$ as every vertex $v \in (N \cap V')$ has a neighbor in $V \setminus N$. Hence by Lemma 2, $\mathbf{tw}(G') \leq \mathcal{O}(\sqrt{(1+\epsilon)k}) = \mathcal{O}(\sqrt{k})$. Now using the constant factor approximation algorithm of Demaine et al. [11] for computing the treewidth of H-minor free graph, we find a tree-decomposition of $G[V']$ of width $\mathcal{O}(\sqrt{k})$ in time $n^{\mathcal{O}(1)}$. Finally, the dynamic programming algorithm mentioned in Lemma 3 runs in time $2^w n^{\mathcal{O}(1)}$ on graphs of treewidth w and hence our algorithm has running time $2^{\mathcal{O}(\sqrt{k})} n^{\mathcal{O}(1)}$. □

4 Partial dominating set problems

In this section we consider PARTIAL r-DOMINATING SET problem. We first modify the Lemma 5 to prove the following.

Lemma 5. *Let $G = (V, E)$ be a graph and let σ be the ordering of the vertices in non increasing order of their sizes of $N_r(v)$, that is, if $v_i < v_j$ in σ, then $|N_r(v_i)| \geq |N_r(v_{i+1})|$ with ties being broken arbitrarily. Let $G = (V_\sigma, E)$ be an yes instance to P-r-DS, $C = \{u_{i_1}, \ldots, u_{i_k}\}$*

be the lexicographically smallest solution for P-r-DS and $u_{i_k} = v_j$ *for some* j*. Then* C *is a* $2r$*-dominating set of size at most* k *for* $G[V_\sigma^j]$*.*

Proof. Let $N_r(C) = \bigcup_{s \in C} N_r(s)$ be the set of vertices of V_σ^j that are r-dominated by C, and suppose that C is not a $2r$-dominating set of V. Let v_i, $i < j$ be a vertex of V_σ^j that is not $2r$-dominated by C ($v_i \notin N_{2r}(C)$). Then $N_r(v_i) \cup N_r(s) = \emptyset$ for every $s \in C$ as otherwise if for some vertex $s \in C$, the intersection is non empty, then v_i will be $2r$ dominated by s. Let $C' = C - v_j \cup \{v_i\}$, then $|C'| = |C|$, C' is lexicographically smaller than C and $|N_r(C')| \geq |N_r(C)| + |N_r(v_i)| - |N_r(v_j)| \geq N_r(C)$ a contradiction to the choice of C. □

We also need a lemma similar to Lemma 3 which we state below.

Lemma 6 ([7])**.** *Let* G *be an undirected graph such that the treewidth of* G *is at most* w*. Then in time* $(2r+1)^{1.5w} n^{\mathcal{O}(1)}$ *we can find a subset* C *of at most* k *vertices that* r*-dominate the maximum number of vertices of* G*.*

With all these ingredients, the subexponential algorithm for the P-r-DS is very similar to our algorithm for PVC. The only difference is in the while loop where instead of finding a dominating set of size $(1+\epsilon)k$, we find a $2r$-dominating set of size $(1+\epsilon)k$, and in the final step, use the dynamic programming algorithm of Lemma 6 to find a subset C of at most k vertices that r-dominate the maximum number of vertices of G. Thus we have

Theorem 2. *Let* $G = (V, E)$ *a graph that excludes an apex graph* H *as a minor and* k *and* t *be a positive integers. Then in* $2^{\mathcal{O}(r(\log r)\sqrt{k})} n^{\mathcal{O}(1)}$ *time we can determine whether there exists a subset* $C \subseteq V$ *of size at most* k *such that* $|N_r(C)| \geq t$*.*

5 Conclusion

We have given the first subexponential algorithms for Partial Vertex Cover and Partial r-Dominating Set problems on planar and apex minor free graphs, answering an open problem in [1]. Our results were based on a simple but powerful observation relating lexicographically least solutions and r-dominating sets of size at most k. This allowed us to significantly improve the running time of several algorithm presented in [1] in an elegant way. Through this process, we have also expanded the list of problems tractable using the irrelevant vertex argument and it would be nice to apply this technique for other problems in planar and other classes of sparse graphs.

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