XML Schema, UNIX Grep and Automata

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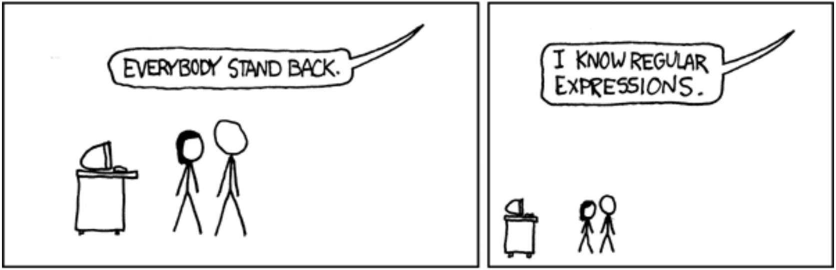
From `xkcd.com` (1)

From xkcd.com (2)



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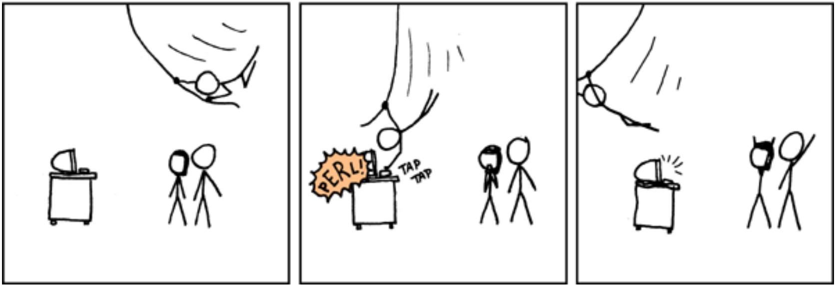


Applications of Regular Expressions

- Searching (UNIX grep, editors)
- Programming language compilers (Lexical Analyzers, e.g. flex)
- Document formats (XML Schema, SGML)

Usual Regular Expressions

Usual Regular Expressions:
$r ::= r + r \mid r \cdot r \mid r^* \mid \Sigma \mid \epsilon \mid \varnothing$
Σ alphabet

Regular Languages

$L(r)$: the regular language denoted by r

- $r \in \Sigma \cup \{\epsilon\} \Rightarrow L(r) = \{r\}$
- $L(r_1 + r_2) = L(r_1) \cup L(r_2)$
- $L(r_1 \cdot r_2) = L(r_1) \cdot L(r_2)$
  where $L(r_1) \cdot L(r_2) = \{w_1 \cdot w_2 \mid w_1 \in L(r_1) \wedge w_2 \in L(r_2)\}$
  and $\epsilon \cdot w = w \cdot \epsilon = w$
- $L(r^*) = \bigcup_{0 \leq i} L(r)^i$
  where $A^0 = \{\epsilon\}$ and for $i > 0$, $A^i = A \cdot A^{i-1}$)
- $L(\varnothing) = \varnothing$

Regular Expressions with Numerical Constraints

- $r ::= r + r \mid r \cdot r \mid r^{\mathbf{N}..\mathbf{N}} \mid r^* \mid \Sigma \mid \epsilon \mid \varnothing$
- $\mathbf{N} = \{1, 2, 3, \ldots\}$
- $L(r_1^{l..u}) = \bigcup_{l \leq i \leq u} L(r_1)^i$
- E.g.: $bab \in L((a+b)^{1..4})$
- In grep: $(a|b)\{1,4\}$
- $L(r^{l..u}) = L(r^l (r+\epsilon)^{u-l})$

XML Schema

- XML document: a tree (labelled, ordered and rooted)
- Labels on children of internal node: a word
- XML Schema: specifies *valid* XML documents:
  - A set of *types*, for each type: a regular expression
  - Assigns a type to each node in valid XML documents
  - the "children-word" must be in the regular expression connected with the type

XML Schema

$(a + b)^{1..4}$ in XML Schema :

```
<xsd:choice minOccurs="1" maxOccurs="4">
<xsd:element name="a" /><xsd:element name="b" />
</xsd:choice>
```

“grepping my mail”

- grep: Searches each line in files for matches to a regular expression
- Examples, GNU grep: '[0-9]{4} [A-Z][a-z]+'
- 4822 matches, 761 MB mail, ca. 50 secs. on 2,4 GHz processor
- Even better: 'Add?ress.*[0-9]{4} [A-Z][a-z]+', ca. 2 secs, 48 matches

Faster, faster!

- match $= \{(r, w) \mid r$ is a regular expression matching $w\}$
- match $\in P$ (Stockmeyer and Meyer, 1973, Kilpeläinen and Tuhkanen, 2003)
- Searching: quadratic number of executions of match-algorithm
- Kilpeläinen and Tuhkanen: matching in quadratic space and time

Deterministic Finite Automata (DFA)

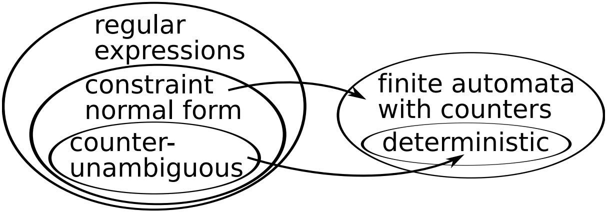
- r regular expression: $\exists$ DFA A: recognizes $L(r)$ in linear time
- prefix $= \{(r, w) | \exists u, v : w = u \cdot v \wedge u \in L(r)\}$
- Searching in word w: $O(|w|)$ executions of prefix
- Deterministic Finite Automata: decides prefix in time linear in $|w|$

Super-polynomial behaviour of grep

- Translating usual regular expressions to DFA is super-polynomial
- From numerical constraints even worse
- GNU grep and Apache Xerces for XML Schema $> 2GB$

- "fast-matcher" for *MATCH*. Polynomial time in regular expression, linear time in word-length
- 1-unambiguity (Brüggemann-Klein, 1992): Polynomial-time construction of DFA from 1-unambiguous regular expressions without numerical constraints
- In XML Schema: Element Declarations Consistent
- No polynomial-time construction from 1-unambiguous regular expressions with numerical constraints known (Equivalent to NP=P, Kilpeläinen, 2004)

- Constraint normal form: polynomial-time decidable subclass of regular expressions
- Counter-unambiguous: polynomial-time decidable subclass of constraint normal form
- Finite Automata with Counters (FAC): polynomial-time construction from Constraint Normal Form
- For Counter-unambiguous: gives deterministic FAC
- Deterministic FAC: linear time matching, quadratic time searching



Comparing greps

- Example: '[0-9]{4} [A-Z][a-z]+'
- 761 MB mail. ca. 70 secs on a $2,4$ GHz processor
- $r = ((0 + \cdots + 9)^{1..2} m((0 + \cdots + 9)^{1..2} s)^{0..60})^{0..60}$
- $2MB$ memory, less than 1 second