INF389 Statistical methods in bioinformatics (H2003) Exercise 2 (Poisson processes)

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Problems marked * are more difficult or technical and can be considered optional.

Problem 1 (4.2 from Ewens & Grant)

Events occur in a Poisson process with parameter λ. Given that 10 events occur in the time period $[0, 2]$, what is the probability that 6 of these events occur in the time period $[0, 1]$? Given that 6 of these events occur in the time period $[0, 1]$, what is the probability that 10 of these events occur in the time period $[0, 2]$?

Problem 2

Let Y be Poisson distributied with parameter λ,

$$P_Y(y) = \frac{e^{-\lambda}\lambda^y}{y!}.$$

By using the well-known series expansion

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

we showed that $\mu_Y = \lambda$.

a) By using the same technique, show that $E(Y(Y-1)) = \lambda^2$.

b) Compute σ_Y^2.

Problem 3

Let Y_1 and Y_2 be independent Poisson distributed with parameters λ_1 and λ_2, respectively. Show that $Y_1 + Y_2$ is Poisson distributed with parameter $\lambda_1 + \lambda_2$.

*Problem 4

In the lectures, we showed that the probabilty distribution for the number of events in a Poisson process after time t, $P_j(t) = \Pr(N(t) = j)$, satisfies the differential equations

$$P_j'(t) = \lambda P_{j-1}(t) - \lambda P_j(t), \quad j = 1, 2, \ldots \tag{1}$$

with the initial conditions $P_j(0) = 0, j = 1, 2, \ldots$. Complete the derivation of the Poisson distribution by showing that the functions

$$P_j(t) = \frac{e^{-\lambda t}(\lambda t)^j}{j!} \tag{2}$$

satisfy (1). (By induction, assume that (2) is true for $j - 1$, substitute this for $P_{j-1}(t)$ in (1), and verify that (2) then satisfies (1)).

*Problem 5 (4.3 from Ewens & Grant)

"Competing Poissons." Suppose that these events occur as described in Section 4.1, but that now each event is of one of k types, labeled types $1, 2, \ldots, k$. The probability that any event is of type i is p_i. Equation (2) continues to give the probability that exactly j events occur in the time period $(0, t)$. Assuming this,

a) Find the (marginal) probability that j_i events of type i occur in the time period $(0, t)$.

b) Find the probability that j_1 events of type 1 occur, ..., j_k events of type k occur in the time period $(0, t)$.

c) Find the conditional probability that j_1 events of type 1 occur, $\ldots j_k$ events of type k occur in the time period $(0, t)$, given that j events altogether occur in this time period. Relate your answer to expression (2.42).