Algorithms for deformable image registration

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Image registration is the task of finding an optimal deformation field for alignment of an input image with a reference image. Useful for comparison of

- Multimodal images (PET, CT, MRI, microscopy, histology)
- Time series (microscopy, DCE-MRI)
For example: Find average intensity value of 1 on the sky. Find the sky in modality 2.
Image registration - definitions

- Input image $f(x, t)$ (template)
- Target image $g(x)$ (reference image)
- Deformation field $u(x, t)$ (displacement field)
- Velocity $v(x, t)$
- Similarity measure $D$
- Regularizer $R$
Image registration - problem definition

Find the solution of

$$\min_u \int_{\Omega} D(x, u) + R(x, u) dx$$

where $D$ can for instance be sum-of-squared differences

$$D = \frac{1}{2}(f(x + u, t) - g(x))^2$$
Methods for image registration

- Affine registration
  - Translation, rotation, scaling, shear
  - Parametric
- Deformable registration
  - All possible deformations $u$
  - Parametric or non-parametric
  - Diffusion
  - Spline
  - Curvature
  - Elastic*
Introduction
Elastic registration
Evaluation of registration
Conclusions

Linear elasticity

- Linear elasticity is valid for small $\nabla u$
- Minimization of total potential energy $V(u)$

$$V(u) = \int_{\Omega} \left( \frac{1}{2} \sigma : \epsilon - f \cdot u \right) dx + \int_{\partial \Omega} t \cdot u \ ds$$  \hspace{1cm} (1)

$\sigma = 2\mu \epsilon + \lambda tr(\epsilon)$, stress tensor
$\epsilon = \frac{1}{2}(\nabla u + \nabla^T u)$, strain tensor
$f$: volume force
$t$: surface force

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Algorithms for deformable image registration
Navier-Lame operator

- Variational operator $\delta V$ leads to Navier-Lame operator
  \[
  \mu \Delta u + (\mu + \lambda) \nabla (\nabla \cdot u) + f = 0
  \]
  Minimization of strain energy $\sigma : \epsilon$

- $f$ is the functional derivative of the similarity measure
- In registration: $f$ is unphysical
  - SSD
  - Cross correlation
  - Normalized gradients
  - Mutual information
  - ...
Cost function sum-of-squared differences (SSD)

- Valid for mono-modal images
- Expecting input and target image to have the same intensity in same location

\[ D_{SSD} = \frac{1}{2} (f(x + u, t) - g(x))^2 \]  (2)
Cost function normalized gradients (NGF)

- Valid for multi-modal images with distinct edges
- Expecting input and target image to have the aligned or co-aligned gradient vectors

Given \( f = f(x + u, t), g = g(x), \)

\[
D_{NGF} = 1 - \left( \frac{\nabla f}{\sqrt{\|\nabla f\|^2 + \eta^2}} \cdot \frac{\nabla g}{\sqrt{\|\nabla g\|^2 + \eta^2}} \right)^2
\]
Numerical implementation of linear elasticity

- Discretize the Navier-Lame operator with central differences
- Obtaining a nonlinear set of equations

\[ Au = (\mu A_1 + (\lambda + \mu) A_2) u = f(u) \]

- \( u \) is a long vector with voxel values (length \( n \times 4 \), \( n \) number of voxels)
- \( A \) is the discretized Navier-Lame operator
- \( A_1 \) is the discretized Laplace operator, \( \Delta u = (\Delta u_1, \Delta u_1) \),
  \[ \frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1,j,k} - 2u_{i,j,k} + u_{i-1,j,k}}{h_x^2} \]
- \( A_2 \) is the discretized convective operator,
  \[ \nabla(\nabla \cdot u) = \nabla(\partial_x u_1 + \partial_y u_2) \]
$A_1$ Laplace operator
$A_2$ Convective operator
Fixed point iterations

- The system $u = A^{-1}f(u) = g(u)$ can be regarded as a fixed point iteration.
- Will converge for a contractive mapping $g$

\[ |g(u) - g(v)| \leq \lambda |u - v| \]

where $\lambda < 1$.

- Iterate until convergence.
Multigrid

- Multigrid is an efficient solver for linear and nonlinear systems of equations
- Low frequencies on a fine level become high frequencies on coarse level
- Smoother: reducing the high-frequency errors by Jacobi or Gauss-Seidel
- Jacobi and Gauss-Seidel efficiently reduce the high frequency errors
- Restriction: Downsampling the residual error
- Prolongation: Interpolating a correction computed on a coarser grid into a finer grid
Generally, let

\[ Au = f. \]

This is equivalent to

\[ Au + Qu = f + Qu \]

leading to

\[ Qu = (Q - A)u + f \]

Iterative scheme:

\[ Qu^{k+1} = (Q - A)u^k + f \]
Multigrid

Note that

\[ Qu^{k+1} = (Q - A)u^k + f \]

is equivalent to

\[ u^{k+1} = u^k + Q^{-1} (f - Au^k) \]  \hspace{1cm} (3)

Updating \( x \) with the error, and \( Q^{-1} \approx A^{-1} \)

Residual equation \( Ae = r \).
Multigrid

- Let $Q = \{a_{ii}\}$ (diagonal elements): Jacobi iterations
- $Du^{k+1} = (L + U)u^k + f$ where $A = D - L - U$
- Matrix free solver easily generated for each $u_{ij}$ from the diagonal matrix $D$
- Guaranteed convergence if $A$ diagonal dominant, $|a_{ii}| > \sum_{j,j\neq i} a_{ij}$
Multigrid

We treat the RHS as constant within a multigrid cycle, $Au = f(u)$.

- Relax $\nu_1$ times on $A^h u^h = f^h$ with initial guess $v^h$
- Compute residual $r^h = f^h - A^h v^h$ and restrict to coarse grid $r^{2h} = l_h^{2h} r^h$
- Solve $A^{2h} e^{2h} = r^{2h}$ (Residual equation)
- Interpolate coarse-grid error to the fine grid by $e^h = l_{2h}^{h} e^{2h}$ and correct the fine-grid approximation $v^h = v^h + e^h$
- Relax $\nu_2$ times on $A^h u^h = f^h$ with initial guess $v^h$
Input image $f$
Target image g
Registered image
Checkerboards $f,g$
Checkerboards registered, g
Deformation field $u = (u_x, u_y)$ (Unit: voxels)
What about larger deformations?

- Does linear elasticity support larger deformation gradients?
Registration of block image, linear elasticity

\[ f \quad g \quad \text{registered} \]

\[ u_x \]

\[ u_y \]
Fluid registration

- In fluid registration the deformation model is a highly viscous fluid deforming
- Fluid registration can therefore handle large deformations
- *Warning*: Can get singular deformations
- Monitor the Jacobian $J = |\nabla u|$, $du = JdU$
Fluid registration

Navier-Lame operator (Navier-Stokes without acceleration and pressure term)

\[ \mu \Delta \nu + (\mu + \lambda) \nabla (\nabla \cdot \nu) + f = 0 \]

Deformation \( u \) is found from material derivative

\[ \nu = \frac{\partial u}{\partial t} + \nu \cdot \nabla u = \frac{Du}{Dt} \]
Registration of block image, fluid registration

\[ f \quad g \quad \text{registered} \]

\[ u_x \quad u_y \]

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Evaluation of registration

How to evaluate registration?

- Visual inspection
- Cumulative inverse consistency error
- Control point evaluation

In particular for time series (DCE-MRI)

- Temporal variation
- Deviation to a compartment model
- Analysis of compartment model parameters
- Tools combining visualization with quantification???
DCE-MRI

- Bolus injection of contrast agent
- Gadolinium based
- Longer T1 times
- Measure the flow of the contrast
- Aim: Estimate glomerular filtration rate (GFR)
DCE-MRI time series
DCE-MRI time series

Sourbron model
Conservation of mass

\[ C = V_P C_P + V_T C_T \]

Rate of change of mass

\[ V_T \frac{dC_T}{dt} = F_T C_T - (1 - f) F_T C_T \]

Control point evaluation

From Hodneland et. al, Normalized gradient fields and mutual information for motion correction of DCE-MRI time series, Proc. ISPA (2013)
Control point evaluation

<table>
<thead>
<tr>
<th>Registration method</th>
<th>$\mu(C_1)$</th>
<th>$\mu(C_2)$</th>
<th>max($C_1$)</th>
<th>max($C_2$)</th>
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<tbody>
<tr>
<td>Unregistered</td>
<td>3.51</td>
<td>3.99</td>
<td>6.12</td>
<td>7.57</td>
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<td>Affine</td>
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<td>1.61</td>
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<td>0.74*</td>
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<td>MIF</td>
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<td>1.11</td>
<td>2.61</td>
<td>1.75*</td>
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</table>
Blurriness

- Raw image sequence
- Affine
- NGF
- MIE
- MIF
## Temporal variation

<table>
<thead>
<tr>
<th>Subject/l,r</th>
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<th>Affine</th>
<th>NGF</th>
<th>MI</th>
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<tbody>
<tr>
<td>1/l</td>
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<td>2.71</td>
<td>2.78</td>
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From Hodneland et al, Normalized gradient fields for motion correction of DCE-MRI time series, In revision, CMIG
Evaluation of (DCE-MRI) time series
Evaluation of (DCE-MRI) time series
T1-w anatomy
DTI-FA - distortions from Eddy currents
After affine registration

Checkerboard before fluid registration
After fluid registration

Checkerboard after fluid registration
T1-w anatomy
After affine registration
After fluid registration
Conclusions

▶ Elastic image registration attempts to mimic the deformation of a deformable solid
▶ Fluid registration attempts to mimic the flow of a high viscosity fluid
▶ Image registration is a versatile tool using various models and similarity measures
▶ Challenges:
  ▶ Large deformations
  ▶ Missing structural information
  ▶ Evaluation
▶ Still, image registration can account for many artifacts arising from motion or geometrical distortions