Roles in Networks

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Roles in Networks

- Motivation for work:
  - Let topology define network roles.
  - Work by Kleinberg on directed graphs, used topology to define two types of roles: ”authorities” and ”hubs”. (Each node gets a score for each role)
- In the following we will only consider undirected networks.
  - In this case authority and hub scores become the same: ”eigenvector centrality” (EVC) scores
- Our goals:
  - Use topology (via EVC) to define natural clusterings in a graph, and boundaries between the clusters
  - Roles will include ”heads” of clusters, other cluster members, and boundary nodes and links

Our starting point: Eigenvector centrality (EVC)

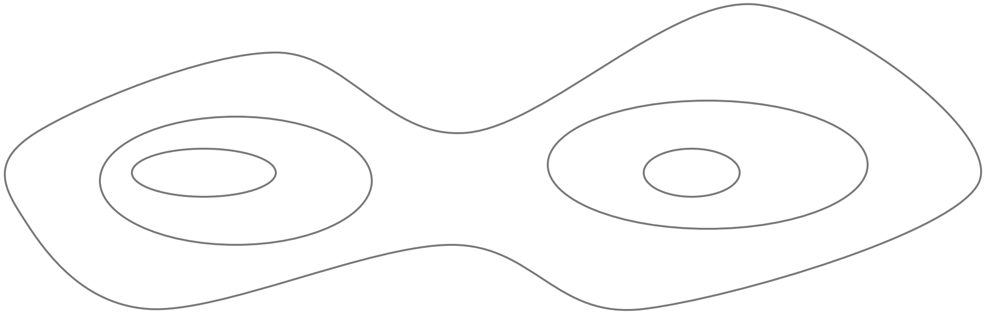
- How central is a node? It's not just how many nodes you know (number of neighbors), but also how central they are, that determines how central you are:

$$e_i \propto \sum_{j=nn(i)} e_j \qquad \text{or} \qquad e_i = \lambda^{-1} \sum_j A_{ij} e_j$$

EVC e_i is thus defined as the *i'th* component of the principal eigenvector of the graph adjacency matrix A.

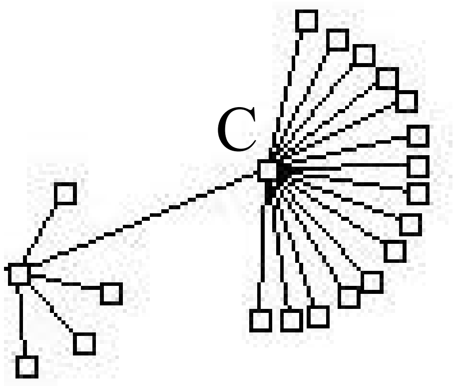
A property of EVC

- Eigenvector centrality is ’smooth’ over the graph.
- Hence, the eigenvector centrality can be viewed as a (smoothly varying) height measure on each network node.
- Topographic view:



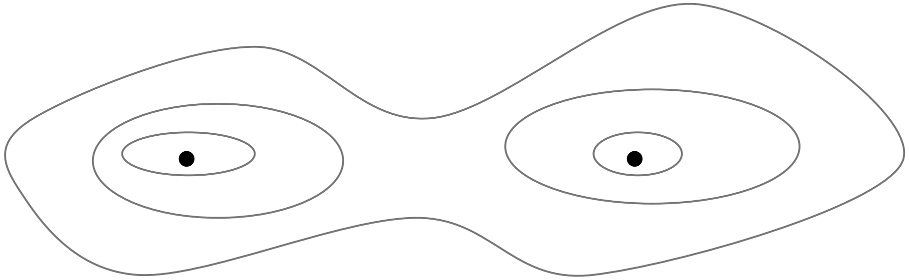
Network role: Centers

- A node is defined as a Center when it is a local maximum in the eigenvector centrality.



Network role: Member of a Region

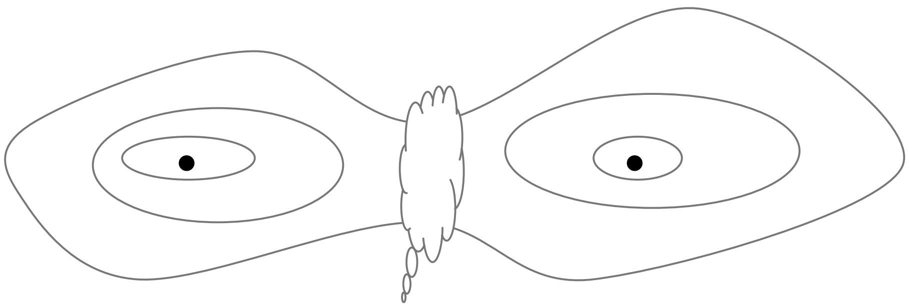
- Idea - Consider the topographic view of EVC:



- Need an unambiguous rule to place nodes in Regions, which are associated with Centers.
- Example rules:
  - Distance from Centers: closest Center determines your Region.
  - Topography: follow steepest-ascent path to find your Center.

Border between regions

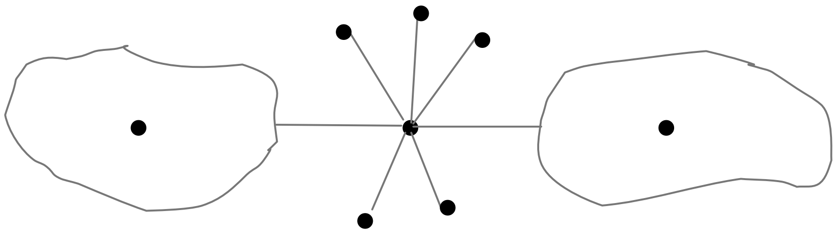
- Topographic picture:



- The Border consists of nodes for which the unambiguous rule gives more than one answer.
- Example rules:
  - Distance: Border set is defined as nodes at equal distance from two or more centers. (So far we have only tried this one.)
  - Topography: Border nodes have more than one steepest path, leading to more than one Center (a rare event)
  - NB: in either case, border links can be defined: they connect two regions

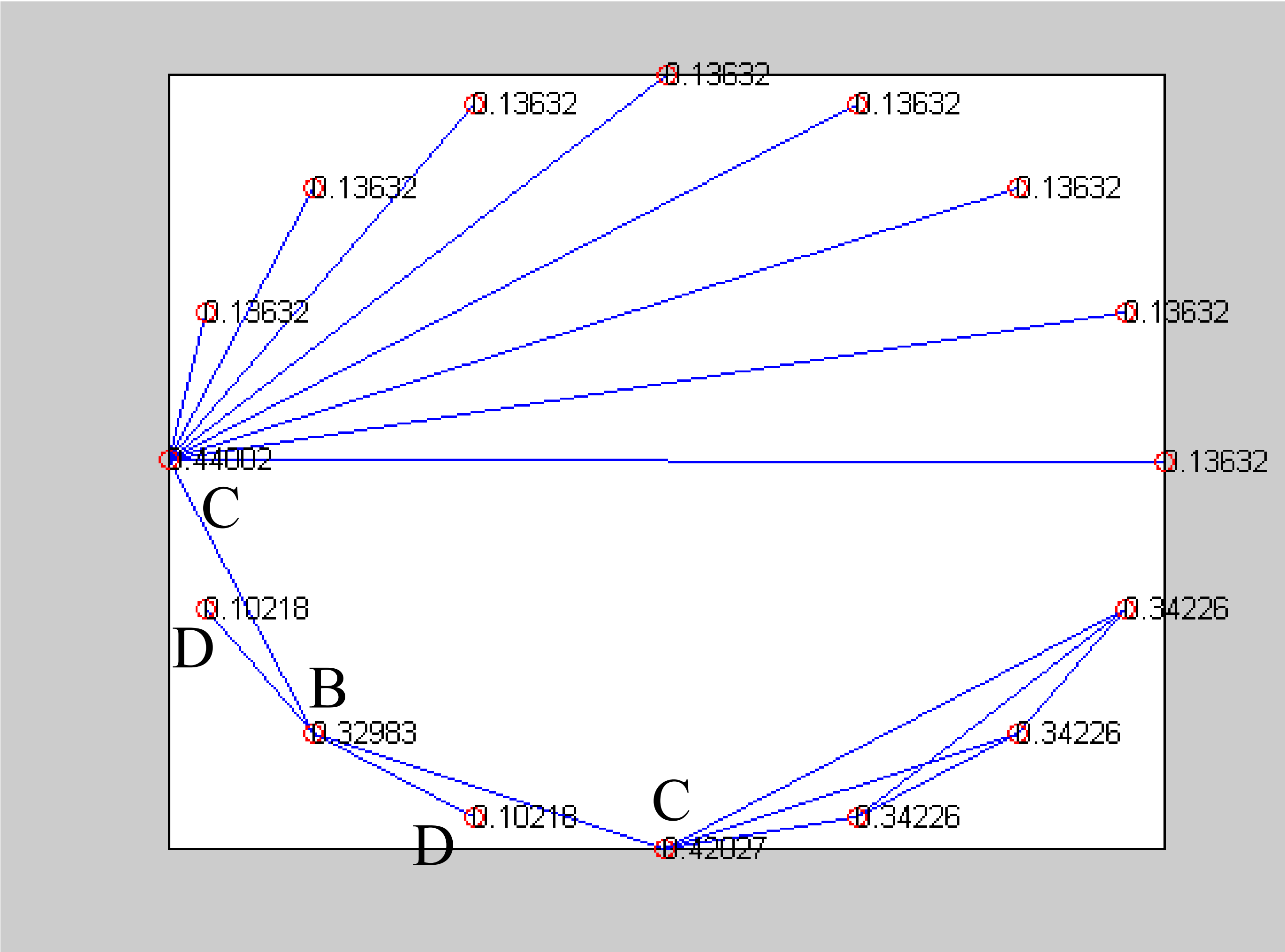
Other network roles: Bridges and danglers

- Using the distance rule to define the border we often get the following:

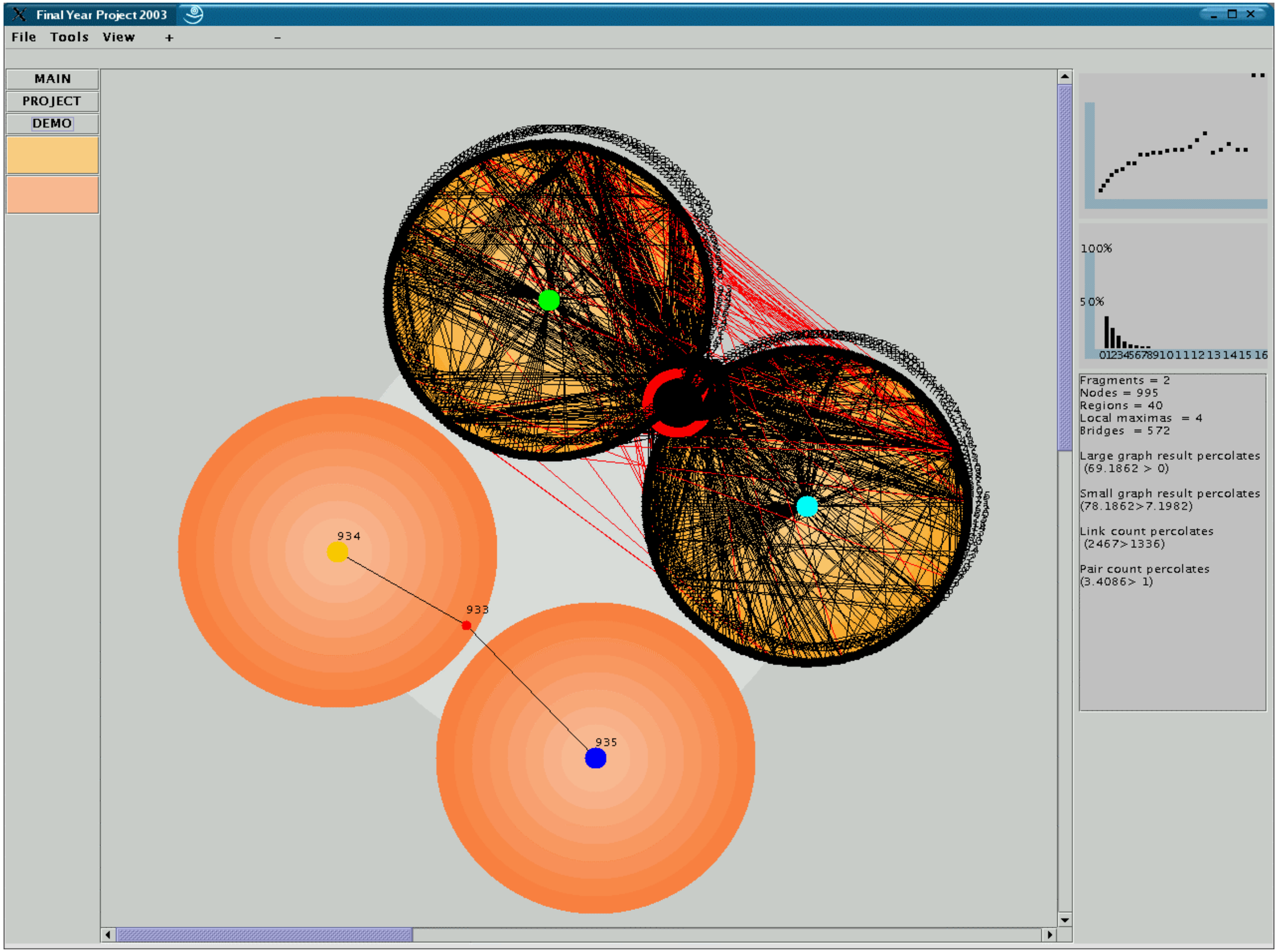


- Based on this we can define <u>two additional network roles</u>:
  - Bridge = Border nodes lying on at least one non-self-retracing path between Centers.
  - Danglers = Border nodes – Bridges.
- These definitions also work for the topographic rule, but these roles are then ’rare’
- With either rule, parallel bridges can occur.

Example network



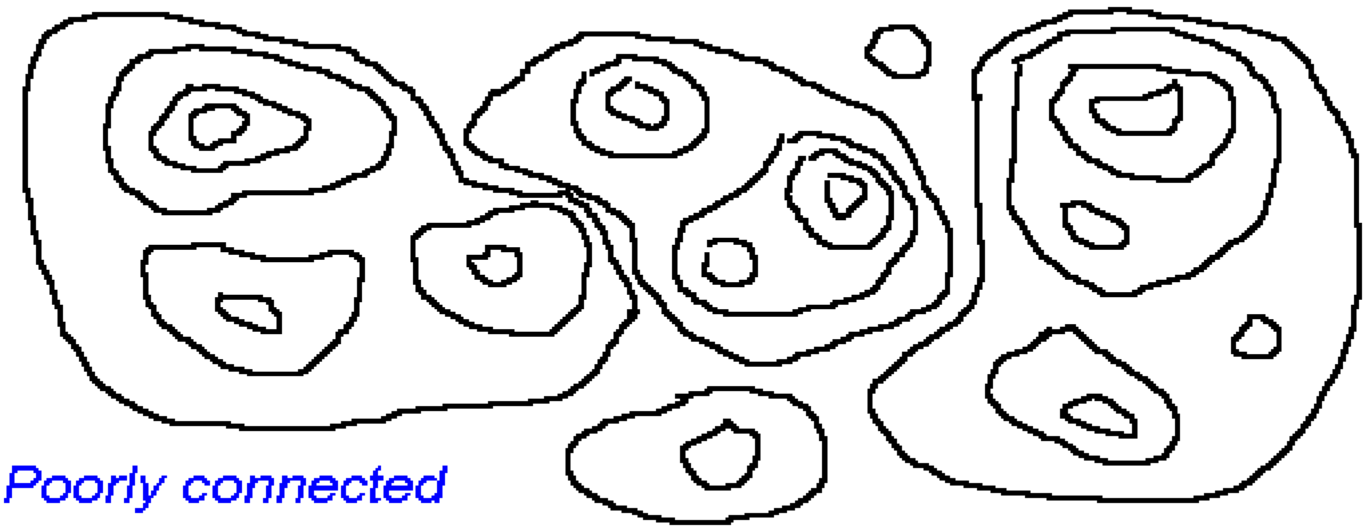
Gnutella visualization using these ideas:



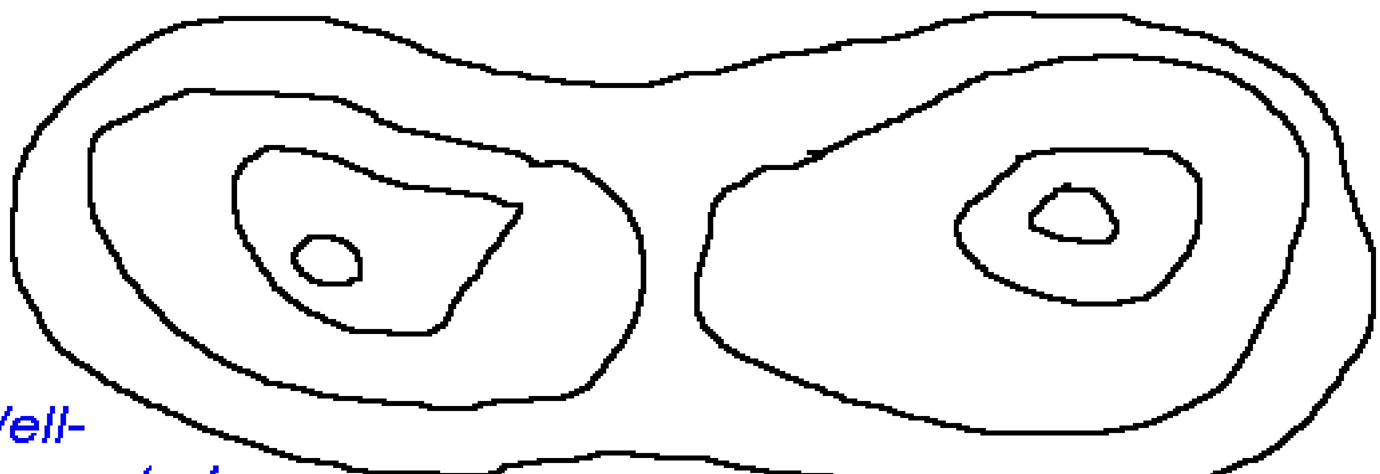
Some thoughts on centrality and well-connectedness

- Conjecture: a well-connected network will have relatively few Centers
- Conjecture: in a well-connected network, there will be many bridges (links and nodes) connecting the few Centers
- To test these conjectures, we need an independent definition of well-connectedness
- Power-law graphs are well-connected, in the sense that they are robust (do not fragment) against the loss of randomly chosen nodes
- Our Gnutella example is a power-law graph, and conforms to our two conjectures above
- Room for more work here!

Thoughts, in pictures



Poorly connected



Well-connected

Summary and future directions

- We have a natural way for decomposing an undirected graph into separate clusters (Regions)
- We have found four distinct network roles in such graphs: Centers, Region members, Bridges, Danglers
- We can also define ”edges” of Regions (nodes touching bridge links, and nodes linked to bridge nodes)
- We have ideas for generalizing this approach to directed graphs
- What’s the use?
  - Helps to understand the structure of a network
  - Eg, gives us measures of well-connectedness ➔ robustness
  - Serves as a guide for visualizing the network in 2D
  - And there must be even more….