Diameter and center computations in networks

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Nordic Network on Algorithms, Istanbul, March 2009

Schedule

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Diameter Computations on Graphs

Before I forgot

This is joint work with D. Corneil, V. Chepoi, F. Dragan, B. Estellon, M. Latapy, C. Magnien, C. Paul,Y. Vaxes

Basics Definitions

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Let G be an undirected graph :

- $exc(x) = max_{y \in G}\{distance(x, y)\}$ excentricity
- $diam(G) = max_{x \in G}\{exc(x)\}$ diameter
- $radius(G) = min_{x \in G}\{exc(x)\}$
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First consequences of the definitions

distance computed in # edges
diameter : Max Max Min
radius : Min Max Min

Trivial bounds

For any graph G :
$radius(G) \leq diam(G) \leq 2radius(G)$ and $\forall e \in G$,
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- If G is a path of length 2K, then $diam(G) = 2k = 2radius(G)$, and G admits a unique center, i.e. the middle of the path.
- If $radius(G) = diam(G)$, then $Center(G) = V$. All vertices are centers (as for example in a cycle).

If $2.radius(G) = diam(G)$, then *roughly* G has a tree shape (at least it works for trees).
But there is no nice characterization of this class of graphs.

Diameter

Applications

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2. Many distributed algorithms can be analyzed with this parameter (when a flooding technique is used to spread information over the network or to construct routing tables).
3. Verify the small world hypothesis in some large social networks, using J. Kleinberg's definition of small world graphs. Then compute the diameter of the Internet graph, or some Web graphs, i.e. massive data.

FAQ

Usual questions on diameter, centers and radius :

- What is the best Program (resp. algorithm) available ?

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- What is the complexity of diameter, center and radius computations ?
- How to compute or approximate the diameter of huge graphs ?
- Find a center (or all centers) in a network, (in order to install serveurs).

Some notes

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- Any algorithm that computes all distances between all pairs of vertices, complexity $O(n^3)$ or $O(nm)$. As for example with $|V|$ successive Breadth First Searches in $O(n(n+m))$.
- Best known complexity for an exact algorithm is $O(\frac{n^3}{log^2 n})$

First theorem

Camille Jordan 1869 :

A tree admits one or two centers depending on the parity of its diameter and furthermore all chains of maximum length starting at any vertex contain this (resp. these) centers.

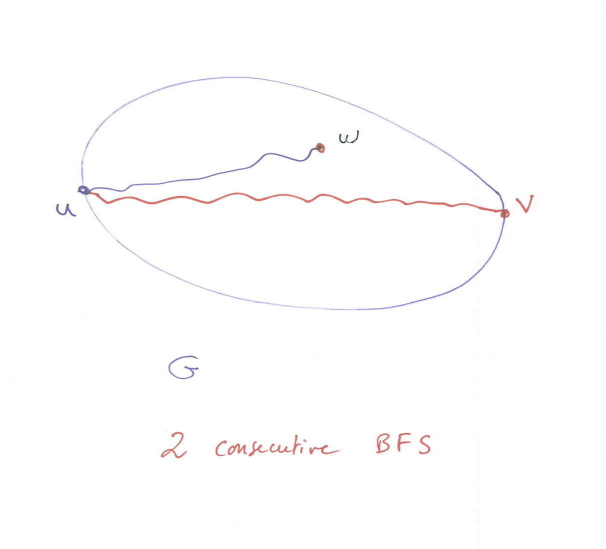
And $radius(G) = \lceil \frac{diam(G)}{2} \rceil$

1. Let us consider the procedure called : 2 consecutive BFS

Data: A graph $G = (V, E)$
Result: u, v two vertices
Choose a vertex $w \in V$
$u \leftarrow BFS(w)$
$v \leftarrow BFS(u)$

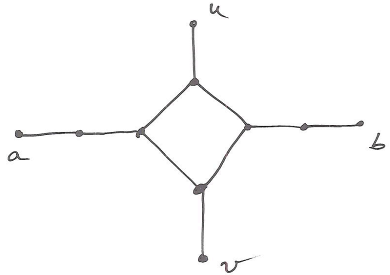
Where BFS stands for Breadth First Search.
Therefore it is a linear procedure

Intuition behind the procedure



- Folklore
  If G is a tree, $diam(G) = d(u, v)$
  Easy using Jordan's theorem.

Unfortunately it is not an algorithm !



Certificates for the diameter

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To give a certificate $diam(G) = k$, it is enough to provide :

- a chain $[x, y]$ of length k with no chord.
- a subgraph $H \subset G$ with $diam(H) = k$,
  H may belong to a class of graphs on which diameter computations can be done in linear time.

Experimental results : M.H., M.Latapy, C. Magnien 2007

Randomized BFS procedure

Data: A graph $G = (V, E)$
Result: u, v deux vertices
Repeat α times :
Randomly Choose a vertex $w \in V$
$u \leftarrow BFS(w)$
$v \leftarrow BFS(u)$
Select the vertices u_0, v_0 s.t. $distance(u_0, v_0)$ is maximal.

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   $exc(u_0) \leq diam(G)$ i.e. a lower bound of the diameter.
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3. Spanning trees given by the BFS.

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- 2 millions of vertices, diameter 32 within 1

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- How can we explain the success of such a method ?
- Due to the many counterexamples for the 2 consecutive BFS procedure. An explanation is necessary !

2 kind of explanations

The method is good or the data used was good.

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Partial answer

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Restriction

First we are going to focus our study on the 2 consecutive BFS.

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3. Corneil, Dragan, Kohler 2003 show for 2 consecutive BFS :
$d(u, v) \leq diam(G) \leq d(u, v) + 2$

A nice algorithmic problem on subset families

Let X be a finite set, and $\mathcal{F}$ be a family of subsets of X.

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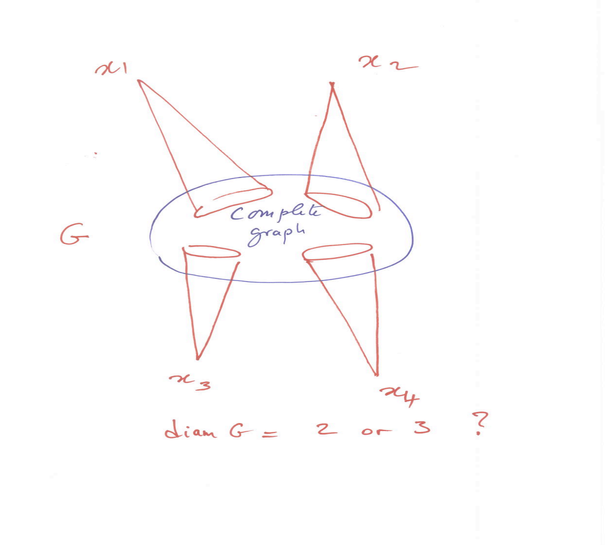
- linear i.e. in $O(|X| + |\mathcal{F}| + \sum_{\mathcal{S} \in \mathcal{F}} |\mathcal{S}|)$

Disjoint sets problem and diameter of split graphs



K-chordal graphs

[CDK'03]

If G is k-chordal (i.e. G does not contain any cycle of length $\geq k$), then the two consecutive BFS allow to find a vertex x tel que $ecc(x) \geq Diam(G) - \lfloor k/2 \rfloor$.
where x is the middle of $[u, v]$.

- Diameter definition can be extended to any subset A of vertices and let us denote by
  $diam(A) = max_{x,y \in A}\{d_G(x,y)\}$
  By convention $diam(\emptyset) = 0$.
  Warning : distances are computed in the whole graph, not inside the subgraph $G(A)$!

Tree decomposition

Let us recall that a graph $G = (V, E)$ has a tree-decomposition $D = (S, T)$ if $S = \{S_1, S_2, \ldots, S_h\}$ is a collection of subsets of V, called bags, T a tree whose vertices are elements of S such that :

(0) The union of elements in S is V

(i) $\forall e \in E, \exists i \in I$ with $e \in G(S_i)$.

(ii) $\forall x \in V$, the elements of S containing x form a subtree of T.

- An important property of tree decompositions :
  Let S_1S_2 be an edge of T (joining the two bags S_1 and S_2), let T_1 and T_2 be the subtrees of T obtained by r removing the edge S_1S_2. Then, $I = S_1 \cap S_2$ separates (i.e. is a separator in G) vertices of T_1-I from T_2-I.

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- A very important notion in graph theory (B. Courcelle, P. Seymour, N. Roberston ......)

Treelength by Dourisbourne and Gavoille 2003

Let us consider a new graph parameter, denoted by $treelength(G)$ and defined as follows :
$Treelength(G) = min_{over\ all\ D}\{max_{S\ bag\ of D}\{diam(S)\}\}$
In other words, for treewidth one measures the maximum size of a bag, as for treelength one measures the maximum diameter of a bag.

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- Idem for distance hereditary graphs.

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- Easy to show $Treelength(G_{n,m}) \leq min\{n, m\}$

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- For a clique K_n $Treewidth(K_n) = n\text{-}1 > Treelength(K_n) = 1$ and for a cycle of length n $Treewidth(C_n) = 2 < Treelength(C_n) = n/3$

- $Treelength(G) \leq k$ iff
  there exist a chordal completion H of G in which all maximal cliques C satisifies $diam_G(C) \leq k$.

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  there exist a chordal completion H of G in which all maximal cliques C satisifies $diam_G(C) \leq k$.
- Computing Treelength is NP-hard, D. Lokshtanov MFCS 2007 !

- Treelength was introduced by Y. Dourisboure and C. Gavoille in order to capture the structure of a network. And they described some efficient routing protocols on networks having bounded treelength.

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- Chepoi, Dragan, H., Estellon, Vaxes 2007
  If G has Treelength k, then
  $2(radius(G) - k) \leq diam(G) \leq 2radius(G)$
  If G has Treelength k, then G is $k/2$-chordal.

Hyperbolic metric spaces

Gromov's 1987 Definition

A graph is δ-hyperbolic iff :
For every four vertices u, v, w, z they are 3 distances (3 matchings)
d(u,v)+d(w,z) and d(u,w)+d(v,z) and d(u,z)+d(v,w)
the two maximal values differ by at most 2δ

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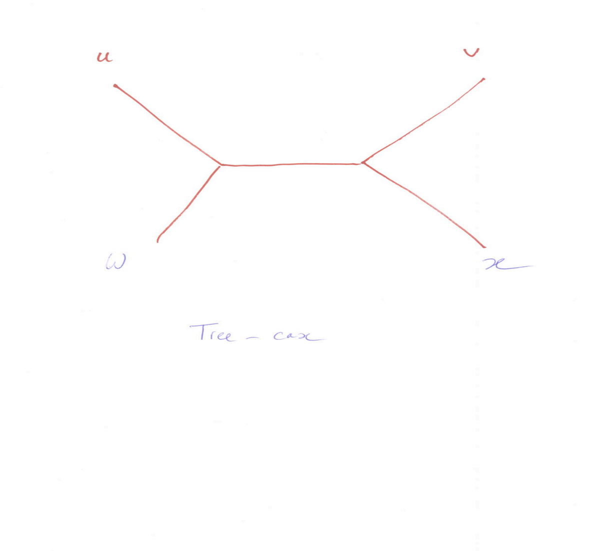
the two maximal values differ by at most 2δ

δ-hyperbolicity can be easily computed in ($O(n^4)$), therefore is polynomial.

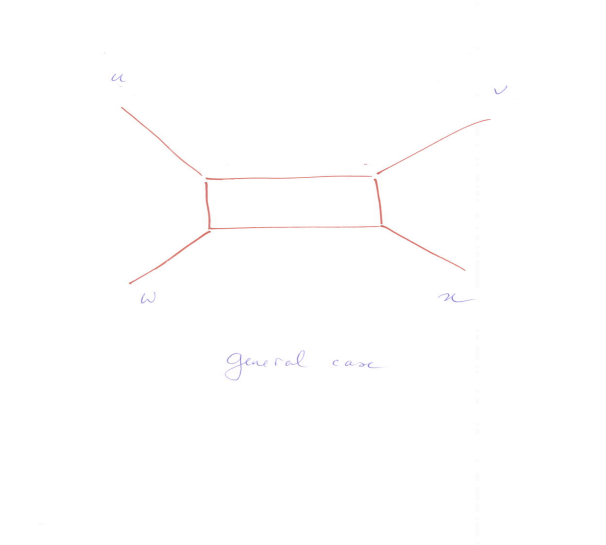
Misha Gromov from his web page !

Misha Gromov

For a tree $\delta = 0$



For general graphs



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2. δ-hyperbolicity is a kind of measure via metric distances to a tree.
3. Many usual graph classes have small δ-hyperbolicity.

- $\delta(K_n) = 0$, $\delta(G) = 0$ iff G is a cactus of cliques.

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- G chordal implies $\delta(G) = 1$.
- Chepoi characterized graphs such that $\delta(G) = 1$.

Last results Chepoi, Dragan, Estellon, H., Vaxes 2008

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If G has treelength k then G is k-hyperbolic

For a δ-hyperbolic graph, $d(u,v) \geq diam(G) - 2\delta$
and $diam(C(G)) \leq 4\delta + 1$

Metric spaces again

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- Relationships with our 2 BFS method and some method in computational geometry to otain the center of a polygon.

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- or closed to a tree in a metric way for δ-hyperbolicity
- then we can bound the behavior with an additive constant of the 2-consecutive BFS.
- We should go further ....

First Attempt

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- So the probabilistic analysis on the 2-sweep heuristic remains to be done !

Centers for chordal graphs

- Let C be the set of all centers of G, if G is chordal then $G(C)$ is m-connected, $diam(G(C)) \leq 3$ and $radius(G(C)) \leq 2$.

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- Chepoi and Dragan ESA 1994 used this property to build a beautiful linear time algorithm to find a center in a chordal graph.
- They generalized this technique to other classes of graphs : HDD-free . . .
  For k-chordal, $diam(G(C)) \leq k$.

Center computations

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- Can you compute $C(G)$ in linear time for chordal graphs ?

Back to "applications"

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2 graphs

Internet Topology Data Kit (ITDK) graph of the routing machines
Treedwidth ≥ 234, Treelength≤ 10, Diameter$=19$,
δ-hyperbolicity$=3$ (but for 96 % of the vertices its value is 1)
Autonomus System Internet Topology (AS-level) graph, a smaller graph
Treedwidth ≥ 82, Treelength ≤ 6, Diameter$=10$, δ-hyperbolicity$=2$ (but for 98 % of the vertices its value is 1)

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- It is far beyond the scope of our knowledge to compute treewith of such graphs
- δ-hyperbolicity seems to be an interesting parameter for networks.
  (already noticed by R. Kleinberg and others).

More on Gromov's metric spaces

- Same theorems for discrete metrics and usual ones.

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- Dynamic maintenance of diameter

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- Provide efficient computations or approximations for the δ-hyperbolicity.
- Extend the probabilistic analysis of the 2 consecutive BFS procedure.
- We still do not know precisely if the computation of the diameter requires the computation of all distances.
- Non uniform diameter (weighted graphs)
- Dynamic maintenance of diameter
- Distributed versions for networks.

Suboptimal algorithms

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- The idea is to only parse part of the input to obtain evaluations of the property you want to compute. Also known as property testing.
- When can you say that a graph is not bipartite wiht high probabilty, without considering the whole graph.
  Many Noga Alon's papers on these kind of questions.

The maximal flow problem : a good case study

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- Many polynomial algorithms available, but not easy to use on massive graphs.
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- Strangely such approximation algorithms are known for many NP-complete problems !

What is known

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- It seems to be easier to find easily a min cut than a flow.

Hints to attack the problem

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- Use PageRank to build a kind of preflow . . .
- Need for 20 years of hard research ?
- Or just 20 years to find the right idea ?

This approach can be used for all polynomial problems for graphs for which the exact algorithms have a complexity quadratic or $O(n.m)$.
Somehow the complexity barrier of the boolean matrix multiplication.

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Thank you for your attention ! !

Merci de votre attention ! !