Automating Coherent Logic
an overview

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Status

  - Graduated: PhD Andrew Polonsky
  - PostDoc: Dag Hovland
- Collaboration (past, present, future)
  - Thierry Coquand (Chalmers)
  - Martin Giese, Bjarne Holen (UiO)
  - John Fisher (CalPolyTech, Pomona)
  - Hans de Nivelle (U Wroclaw)
  - Stefan Berghofer (TUM, Isabelle)
  - Hopefully you (…)
- Necessary: new impulses, initiatives
What has been achieved so far?

- Several CL-provers
- Some provers provide proof objects
- One prover (Geo) competitive in CASC 06-07 (FOF,FNT)
- A family of translations from FOL to CL
- Proof objects for the translation(s)
- Some integration in Isabelle (tactic coherent)
- About a dozen publications in various venues
- Ongoing: clp (RETE, Hovland), coColog (Fisher)
Challenges

- More efficient proof search in CL
- Evaluation of the translations FOL $\rightarrow$ CL
- Extensions with native equality
- Extensions with function symbols
- Tighter integration in proof assistants
Translation form FOL to CL

- Example: Peirce’s Law \(((p \rightarrow q) \rightarrow p) \rightarrow p\)
- Variables \(T, F : Prop \rightarrow Prop\) ‘freezing the arguments’, later expressing true and false
- Coherent theory:
  1. \(F(((p \rightarrow q) \rightarrow p) \rightarrow p)\)
  2. \(F(((p \rightarrow q) \rightarrow p) \rightarrow p) \rightarrow T((p \rightarrow q) \rightarrow p) \land F(p)\)
  3. \(T((p \rightarrow q) \rightarrow p) \rightarrow F(p \rightarrow q) \lor T(p)\)
  4. \(F(p \rightarrow q) \rightarrow T(p) \land F(q)\)
  5. \(T(p) \land F(p) \rightarrow \bot\)
- Refute in CL, substitute \(T = \lambda p. p\) and \(F = \lambda p. \neg p\) (on blackboard)
Example, continued

- $T((p \to q) \to p) \to F(p \to q) \lor T(p)$ was a choice
- Why not $T((p \to q) \to p) \land T(p \to q) \to T(p)$?
- Then you would need $F(p \to q) \lor T(p \to q)$ ...
- Sometimes, no disjunctions needed: $p \to q, p \vdash q$
- Polonsky’s translation looks for the optimal polarities
Elimination of function symbols (de Nivelle)

- Unary predicates for constants: \( C(x) \) for \( c = x \), axiom \( \exists x. C(x) \)
- Binary predicates for unary functions: \( F(x, y) \) for \( f(x) = y \), axiom \( \exists y. F(x, y) \)
- And so on, unicity is not required!
- Example: for constants, \( a = b \) becomes \( A(x) \leftrightarrow B(x) \)
- Equality almost vanishes: only needed between variables
- A combinatorial puzzle: each \( n \in \mathbb{N} \) is either red or green but not both. For each \( n \in \mathbb{N} \), if \( n \) is red then \( n + 2 \) is green else \( n + 1 \) is red. Is this possible?
Puzzle formalized in CL with functions

- $r(x) \rightarrow g(f(f(x)))$
- $g(x) \rightarrow r(f(x))$
- $r(x) \lor g(x)$
- $r(x) \land g(x) \rightarrow \bot$
- Domain non-empty!
Puzzle, functions eliminated

\[\exists y. F(x, y)\]
\[r(x) \land F(x, y) \land F(y, z) \rightarrow g(z)\]
\[g(x) \land F(x, y) \rightarrow r(y)\]
\[r(x) \lor g(x)\]
\[r(x) \land g(x) \rightarrow \bot\]
Proof and recovery of proof object

- If necessary, translate FOL $\rightarrow$ CL
- Refute with prover
- Proofs are valid for all relations $F(x, y)$
- In particular for $F(x, y) \equiv (f(x) = y)$, substitute
- $\exists y. f(x) = y$ is a tautology
- $g(x) \land f(x) = y \rightarrow r(y)$ is equivalent to $g(x) \rightarrow r(f(x))$
- Similarly for all other axioms
- Proof of original formula is obtained
Queueing depth-first

- Observation: $\exists y. F(x, y)$ is harmful for depth-first search
- Recommended order for depth-first search:
  - Horn clauses clauses
  - Disjunctive clauses
  - Existential clauses
  - Disjunctive existential clauses
- Without functions, depth-first terminates for the first two
- Depth-first search not complete for one single existential clause, subtle:
  $$p(a). \ p(b). \ q(b)\rightarrow\text{goal}.$$
  $$p(X), p(Y)\rightarrow \text{exists } U: p(U), q(X), r(Y).$$
- Queueing depth-first: the (disjunctive) existential clauses in a cyclic queue + iterative deepening wrt constants. Complete.
Useful links

- General CL:
  http://www.johnrfisher.net/index.html

- RETE experiments:
  http://code.google.com/p/clp/

- Colog:
  www.csupomona.edu/~jrfisher/colog2012/