Complexity Classes

P - class of computational problems that can be solved "quickly" on a classical computer.

NP - class of problems which have solutions which can be quickly checked on a classical computer.

e.g. finding the prime factors of n.

- maybe this problem is not in P

... but it is certainly in NP

Clearly,

$$P \subset NP$$

Open problem,

$$P \overset{?}{\neq} NP$$

.... most researchers believe

$$P \neq NP$$

Can QCs quickly solve all problems in NP?

.... no-one knows...

note: factoring is in NP but it is not known whether factoring is NP-complete.

One Negative Result

No search-based methodology can yield an efficient solution to all problems in NP

.... so this aspect of quantum parallelism cannot be exploited.

PSPACE

problems solvable using:

...... small spatial resources

...... potentially large time resources

Conjecture:

BPP

problems solvable using randomized algorithms in polynomial time, assuming a bounded probability of error.

BPP is considered to be the class of problems efficiently solvable on a classical computer.

Quantum Complexity Classes?

BQP:

Efficiently solvable on a QC with bounded error.

... relationship between BQP and P, NP and PSPACE is unknown.

.... QCs can solve all problems in P efficiently, but there are no problems outside PSPACE that they can solve efficiently.

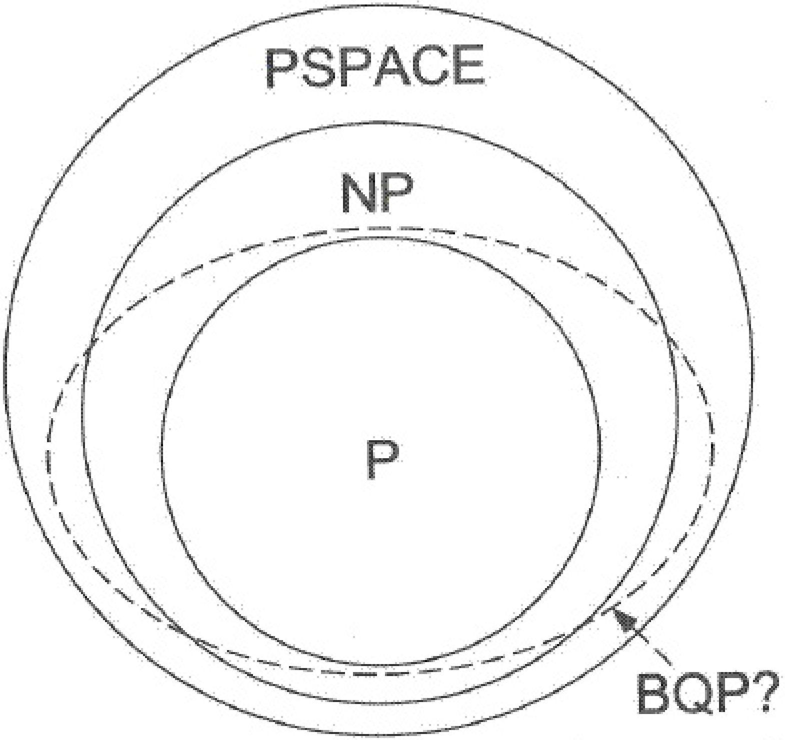


Figure 1.21. The relationship between classical and quantum complexity classes. Quantum computers can quickly solve any problem in **P**, and it is known that they can't solve problems outside of **PSPACE** quickly. Where quantum computers fit between **P** and **PSPACE** is not known, in part because we don't even know whether **PSPACE** is bigger than **P**!

Experimental Quantum Information Processing

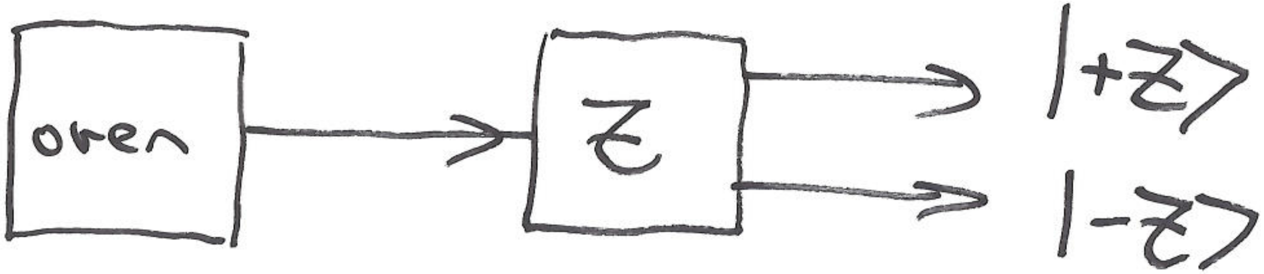
Experimental feasibility?

Stern-Gerlach experiment:

evidence for existence of Natural qubits.

Stern-Gerlach Experiment

Stern (1921) with Gerlach (1922)



Hot hydrogen atoms beamed from an oven through a magnetic field causing deflection either up ($|+z\rangle$) or down ($|-z\rangle$)

Hydrogen atom:

proton with electron orbiting about
atomic axis

atom has magnetic field
(magnetic dipole moment)
oriented about axis.

One would expect deflections to be uniformly random,

but deflections are taken from a discrete set of angles!

Explanation:

Magnetic dipole moments are quantized.

experiment indicates that one, undeflected, beam of atoms would be seen,

.... but ....

two beams are seen....

one deflected up, one down.

To explain these results the notion of

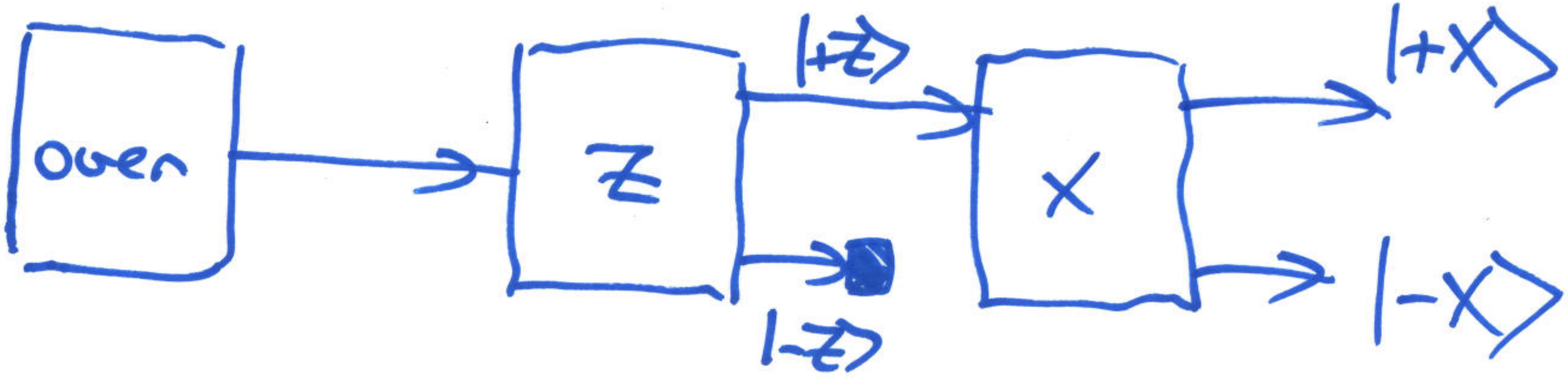
"SPIN"

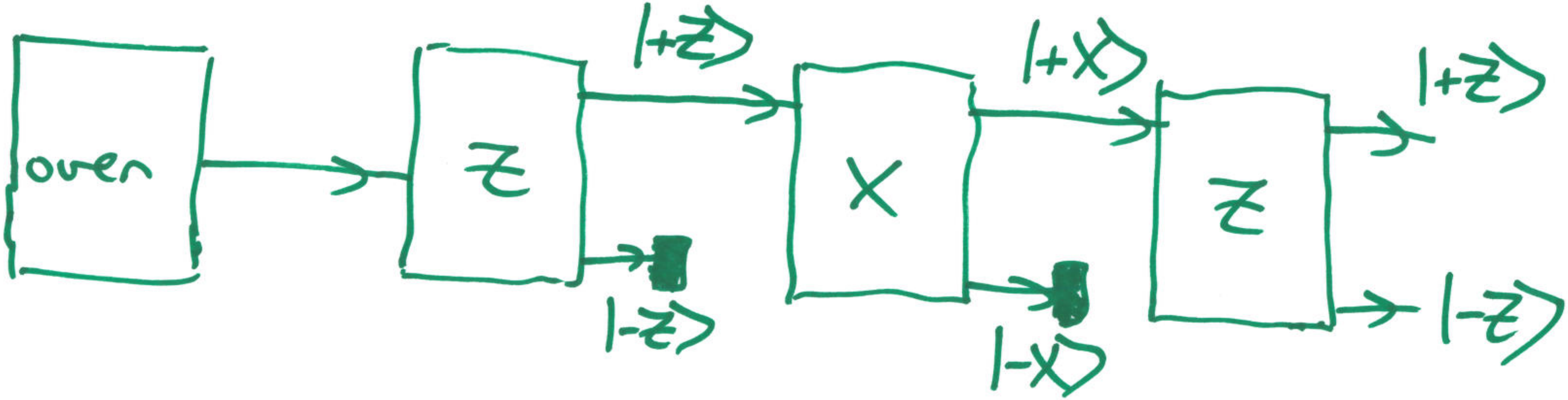
was introduced.

...... a new quantity associated to the electron.

Two More Stern-Gerlach Experiments

…. using apparatus aligned in Z and X directions.





Qubit Model Explains Measurements

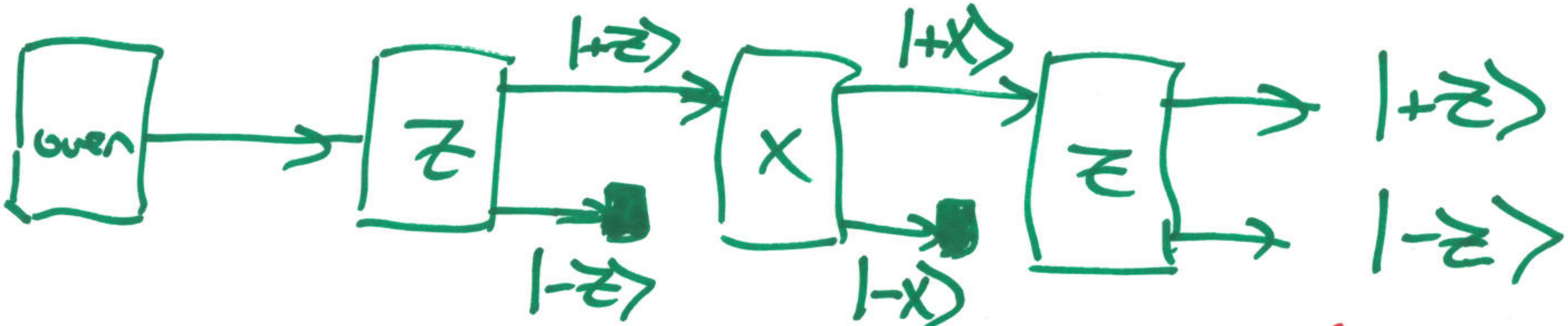
Assign,

$$|+Z\rangle \leftarrow |0\rangle$$
$$|-Z\rangle \leftarrow |1\rangle$$
$$|+X\rangle \leftarrow (|0\rangle + |1\rangle)/\sqrt{2}$$
$$|-X\rangle \leftarrow (|0\rangle - |1\rangle)/\sqrt{2}$$

Z - Stern-Gerlach apparatus measures spin (qubit) in computational basis.

X - Stern-Gerlach apparatus measures spin in $(|0\rangle + |1\rangle)/\sqrt{2}$, $(|0\rangle - |1\rangle)/\sqrt{2}$ basis.

e.g.



$$|+z\rangle = \left(|+x\rangle + |-x\rangle\right)/\sqrt{2}$$

after second experiment, spins are in
state $|+x\rangle$ with pr $\frac{1}{2}$
" $|-x\rangle$ " "

after third experiment, spins are in
state $|+z\rangle$ with pr $\frac{1}{2}$
" $|-z\rangle$ " "

Prospects for Practical Quantum Information Processing

Noise : a fundamental barrier?

Quantum Mechanics : correct?

Threshold Theorem

Provided the level of noise can be pushed down below some threshold then, for a small overhead, quantum error-correcting codes can further reduce noise by an arbitrary amount.

Quantum State Tomography

Determine quantum state of system

.....overcome "hidden" nature of state..

... measure repeated preparations of the same state in different ways.

Quantum Process Tomography

Characterise dynamics of quantum system

e.g.

assess performance of quantum gate or quantum channel.

Simulation of Quantum Systems

..... containing a few "dozen" qubits.

e.g. 50 qubits requires $2^{50} \simeq 10^{15}$ complex values

.... each value requires 256 bits

$\Rightarrow 32 \times 10^{15}$ bytes of information.

Large-Scale Applications?

factoring, discrete log

search

.... any more ideas?

Physical Realizations

Optical : (mirrors, beamsplitters)

single photons?

information stored in photons

advantage - stable

disadvantage - don't easily interact.

Ion Trap: small number of charged atoms in a confined space.

Neutral Atom Trap: uncharged atoms in a confined space

- photons manipulate information stored in atom
- single qubit gates use electromagnetic pulses
- multi-qubit gates via atomic interactions controlled by electromag-pulses
- measurement via "quantum jumps" technique.

Nuclear Magnetic Resonance (NMR)

- information in nuclear spin of atoms in a molecule.
- manipulation via electromagnetic pulses
- huge number ($\sim 10^{15}$) "identical" molecules stored in solution (classical parallelism)

.... large scale causes problems from a quantum perspective...

... sufficiently fine-grained quantum properties exploited by individually addressing differing nuclei in molecule.