Optimization Methods for Pipeline Transportation of Natural Gas

– DISSERTATION –

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December 16, 2010

Agenda

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1. Introduction
   - My PhD research frame
2. Project I:
   - The fuel cost minimization problem
   - Three solution approaches
   - Numerical experiments & results
3. Project II:
   - Flow capacity optimization
   - Key issues of the model
     - Gas specific gravity, g
     - Gas compressibility, z
4. Project III:
   - The line-packing problem
   - Key issues of the model
5. Concluding Remarks

A long history in a nutshell

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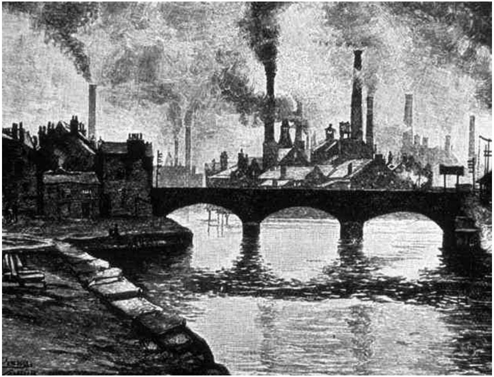
FOSSIL FUELS: Coal, petroleum & natural gas



Rural economy	→	THE INDUSTRIAL REVOLUTION	→	**Urban economy**

A strategic commodity...

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Natural gas (NG) – a colourless, odourless substance primarily composed of hydrocarbons.

Pipeline network systems → A means for transporting NG

My PhD research frame comprises...

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Optimization problems arising in natural gas transmission systems

Project I

Operability on compressor stations

Project II

Flow optimization under variation in volumetric properties

Project III

Optimal line-pack management in gas transportation networks

Project I: Operability on compressor stations

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PROJECT I

Project I: Description of the FCMP

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As we have seen...

- NG, driven by pressure, is transported through pipeline systems.
- Energy & pressure are lost.
- To overcome this loss, compressor stations (CSs) must be turned on.
  (Several identical centrifugal compressor units connected in parallel)
- A significant proportion of the transported gas (3-5%) is consumed by CSs

Project I: Description of the FCMP

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Keeping this consumption at a minimum leads to

THE FUEL COST MINIMIZATION PROBLEM (FCMP)

This has an important environmental dimension and entails a large economic value to the industry.

Related works to the FCMP

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Literature on related works

Jefferson (1960)	the liquid pipeline optimization
Wong and Larson (1968)	1st publication on gun-barrels by DP
Martch and McCall (1972)	tree-shaped networks by DP
Luongo, Gilmour and Schroeder (1989)	cyclic networks by hybrid DP
Percell and Ryan (1987)	GRG on cyclic networks
Lall and Percell (1990)	tree-shaped networks by MINLP
Carter (1998)	cyclic networks by NDP
among others...	

Project I: *An NLP model for the FCMP*

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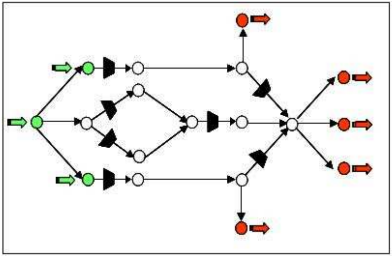
Assumptions (applicable to all projects)

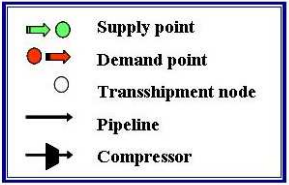
- Gas system is in steady-state
- Network is balanced and directed
- Each parameter is known with certainty
- Constant temperature
- Irreversible flow

Notation

The gas transmission network, variables and parameters

$G = (V, A) \Rightarrow$ **A gas transmission network**





DECISION VARIABLES:

x_{ij} : Mass flow rate in arc (i, j)

p_i : Pressure at node i

PARAMETERS:

P_i^L, P_i^U : Pressure limits at node $i \in V$;

B_i : Net mass flow rate at node $i \in V$.

Project I: *An NLP model for the FCMP*

The FCMP is then formulated as suggested by Wu et. al (2000)

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$$\min \sum_{(i,j)\in A_c} g_{ij}(x_{ij}, p_i, p_j) \quad (1)$$

s.t.:

$$\sum_{i:(j,i)\in A} x_{ji} - \sum_{i:(i,j)\in A} x_{ij} = B_j, \quad \forall j \in V \quad (2)$$

$$(x_{ij}, p_i, p_j) \in D_{ij}, \quad \forall (i,j) \in A_c \quad (3)$$

$$x_{ij}^2 = W_{ij}\left(p_i^2 - p_j^2\right), \quad \forall (i,j) \in A_p \quad (4)$$

$$P_i^L \leq p_i \leq P_i^U, \quad \forall i \in V \quad (5)$$

$$x_{ij} \geq 0, \quad \forall (i,j) \in A \quad (6)$$

Project I: *An NLP model for the FCMP*

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Difficulties are the non-convexity presented by:

$$g_{ij}(x_{ij}, p_i, p_j) = \frac{x_{ij}\left[\left(\frac{p_j}{p_i}\right)^m - 1\right]}{\eta_{ij}}, \forall (i,j) \in A_c,$$ and constraints (3)–(4)

Project I: Solution approaches for the FCMP

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SOLUTION APPROACHES

(How do we propose to solve FCMP in my PhD thesis?)

Two techniques must first be described...

(1) A reduction technique (Ríos-Mercado et al., 2002)

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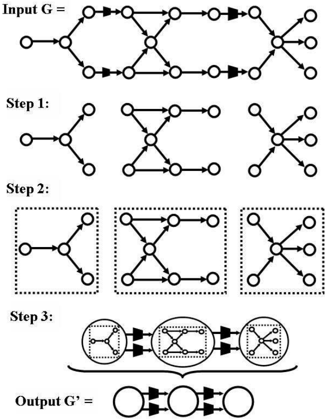

G' = compressor network

(2) The NDP algorithm (Carter, 1998)

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Basic principles of the NDP of Carter

- 1^{st} : **Discretize** $[P_i^L, P_i^U], \forall i \in V$
  Assume τ discretization points
  $(p_i^1, \cdots, p_i^\tau)$ of pressure at node i,
  s.t. $P_i^L \leq p_i^1 < \cdots < p_v^\tau \leq P_i^U$
- For instance, [600,800], $\tau = 20$.
  We would consider only pressures at ten point increments: $600, 610, 620, \cdots, 800$.

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- For instance, $[600,800]$, $\tau = 20$.
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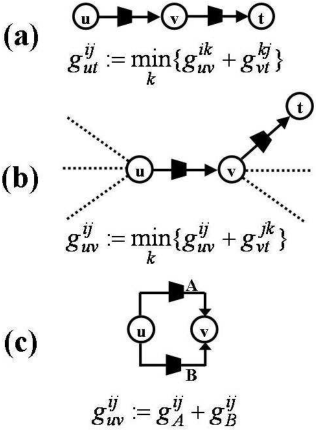
Then NDP consists of

- a sequence of reductions of G' until the resulting graph is a single node.
- 3 types are considered:
  a) Serial, b) dangling and c) parallel.



Project I: *Solving the FCMP*

The 1^{st} solution approach suggested for solving FCMP

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⇒ **A Tabu Search scheme and non-sequential DP**

TS heuristic (Glover and Laguna, 1997) & NDP approach (Carter, 1998)

The 1^{st} *solution approach* (suggested during my master)

Input: $G = (V, A)$

- STEP 0: Obtain a **reduced network** G' of G
- STEP 1: Find a feasible flow x in G'
- STEP 2: Apply **NDP to G' to optimize pressure** p
- STEP 3: Apply a TS heuristic to optimize (x, p) on G'
- STEP 4: Extend the best solution found to G

Output: Feasible flows and optimal pressures for G

Project I: *Solving the FCMP*

Conclusions on *the* 1^{st} *solution approach*

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Advantages:

- ✓ Applicable to *linear, tree-shaped & cyclic networks*
- ✓ Easy to handle *non-convexity*

Project I: *Solving the FCMP*

Conclusions on *the* 1^{st} *solution approach*

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Weakness:

- Limited to sparse networks

Project I: *Solving the FCMP*

Conclusions on *the* 1^{st} *solution approach*

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Weakness:

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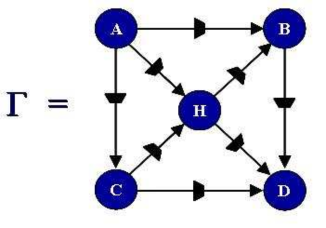
An example of this limitation...



When neither of the reductions (a)–(c) can be carried out, NDP fails!

Project I: *Solving the FCMP*

The 2^{nd} solution approach (Contribution of the PhD thesis)

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(To overcome this weakness...)

A 2^{nd} solution approach

$\Rightarrow$ **A tree-decomposition based algorithm**

Frequency assignment problem via TD (Koster et al., 1999)

A tree-decomposition based algorithm

Devoted to optimize *pressure values*

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TreeDDP Algorithm

Input:	$G' = (V, A_c)$ = Compressor network
	A feasible flow $\boldsymbol{x}$ in G'
Phase 0:	**Apply** NDP to G' $\rightarrow \boldsymbol{\Gamma}$
Phase 1:	**Find** a tree decomposition **T** of $\boldsymbol{\Gamma}$
Phase 2:	**Apply** a DP to T
Output:	Optimal pressure p^*

Phase 1: Tree Decomposition (TD)

(Robertson and Seymour, 1986)

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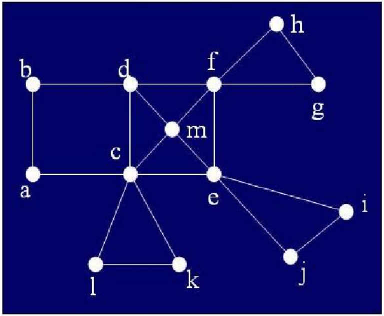
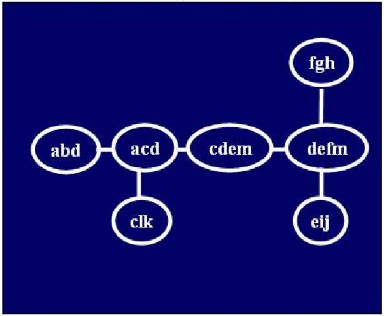
Basically, the main idea of a TD technique

Decompose the problem into a set of connected sub-problems,
where two sub-problems are connected when they share a constraint.





Compressor network Γ $\rightarrow$ Tree decomposition T of Γ

To compute $\mathscr{T} \longrightarrow$ TD technique based on *Maximum Cardinality Search* (MCS)

Phase 2: Dynamic Programming (DP)
(Bellman, 1953)

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Now,

The principles of DP can be applied to T

Components of a global opt. sol. are themselves globally opt.

This completes the 2^{nd} solution approach,
TreeDDP, proposed in this thesis

Project I: Numerical experiments

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Performance of *TreeDDP Algorithm*

Observations for three different mesh sizes:

CPU Time (sec):

	τ = 50	τ = 100	τ = 1000
AVG	0.0	3.2	1535 (0.4hr)
MAX	2.6	34.4	**3623 (1.0hr)**$\star$

- AVG-relative GAP from τ=1000 to τ=50: **42.17%**, τ=100: **5.52%**

Project I: Numerical experiments

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PERFORMANCE OF *TreeDDP Algorithm*

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Conclusion:

τ is crucial for the CPU-time

Numerical Experiments

TreeDDP Algorithm vs. other optimizers

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TREEDDP ALGORITHM VS. BARON AND MINOS

Observations on 16 test instances:

- Instances solved by:
  - **Baron:** 8/16, **Minos:** 11/16, **TreeDDP:** 16/16
- AVG-RI of *TreeDDP* over:
  - Baron: 9.45%, Minos: 11.55%

Project I: *Solving the FCMP*

The 2^{nd} *solution approach*

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Conclusions on the 2^{nd} *solution approach*

Advantages:

- ✓ Applicable to *a wide range of network topologies*
- ✓ *TreeDDP* can be put inside an algorithm that considers flow variables in an outer loop.

Project I: *Solving the FCMP*

The 2^{nd} solution approach

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Weakness:

- ▼ High dependency on *the discretization*, τ

Project I: *Tackling the FCMP*

The 3^{rd} solution approach (Contribution of the PhD thesis)

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(To overcome the weakness presented by TreeDDP...)

The 3^{rd} *solution approach*

$\Rightarrow$ **An adaptive discretization approach**

Based on some **facts:**

- Assessing the number of discretization points, τ, is not a trivial task.
- Large value of τ increases the possibility of finding a good solution.
- **The asymptotic increase in the running time of DP is proportional to τ^d.**

Project I: *Tackling the FCMP*

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Project I: *Tackling the FCMP*

The 3^{rd} solution approach

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Main ideas behind the *adaptive discretization*

- Start with τ as small as possible
- Upgrade τ by a fixed factor until at least one feasible point is found by DP.
- For each solution in a selection of the feasible ones hence found:
  - Define a focus area for the next iteration
- Apply the same procedure to each focus area.

Project I: *Tackling the FCMP*

The 3^{rd} solution approach

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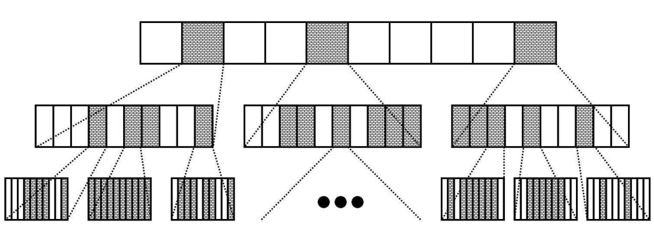
The key point of this approach is

Focus on the neighborhood of some of the feasible solutions found

The search tree



...and explore recursively each solution until

The discretization distance within the focus area drops below a given threshold.

Project I: *Solving the FCMP*

The 3rd solution approach

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Conclusions on the 3rd *solution approach*

Advantages:

- ✓ No assumption is made concerning the sparsity of the network
- ✓ Outperforms *the efficiency of* **previous approaches** (up to 16.3% of RI over a global optimizer)
- ✓ Effective in *solving large networks* (< 60 CPU-seconds in 22 test cases)

Project I: *Solving the FCMP*

The 3rd solution approach

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Weakness:

- ▼ As a heuristic method → *no guarantee of optimality*

Project II

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Let's now move to

PROJECT II

Project II: Key issue

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As observed from the model introduced in Project I

An equation to define the resistance of a pipeline is required.

Fact:

The resistance of the pipeline
are based on current physical gas and pipeline properties

Literature reveals several methods for this purpose

Weymouth eq. (1912) (Osiadacz, 1987),
Panhandle A eq. (1940) and Panhandle B eq. (1956) (Crane, 1982).

Project II: Key issue

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In this project, we also make use of

Weymouth equation: $$x_{ij}^2 = W_{ij}\left(p_i^2 - p_j^2\right)$$

where $$W_{ij} = \frac{d^5}{K g_i z_{ij} T f_{ij} L_{ij}}.$$

Observations:

- W depends on z and g;
  z depends also on g, p, T;
- and g depends on upstream flows because of blending

  based on the well from where the gas originates & degree of processing

Project II: Key issue

UiB

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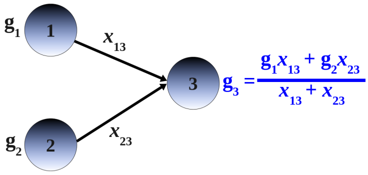

Project II: Key statements

Then, what do we propose in this project?

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Goal of the project

Extend previously suggested models
to bring them closer to physical reality.

Accomplish by

Formulating a model where **not only** x **and** p **but** g **and** z
are defined as state variables in a NG transmission system.

Focus on

Flow maximization of natural gas transmission pipeline systems in steady-state.

Project II: Variability of g and z in pipeline systems

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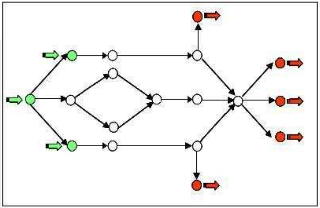
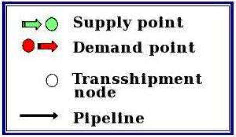
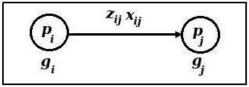
Notation

Decision variables:

- x_{ij} =flow through pipeline $(i,j) \in A$.
- p_i and p_j= upstream and downstream pressures in pipeline $(i,j) \in A$.
- g_i = gas specific gravity at node $i \in V$.
- z_{ij} = gas compressibility factor in pipeline $(i,j) \in A$.







Project II: Modeling *the resistance of the pipeline*

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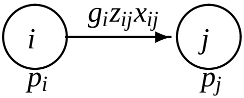
1st key component of the model

By defining $w_{ij} = g_i z_{ij} W_{ij}$,

Weymouth eq. can be written as



$$g_i z_{ij} x_{ij}^2 = w_{ij} \left(p_i^2 - p_j^2 \right) \quad (1)$$

Thus, **(1)** is adopted in the model.

Project II: Balance of g in the system

2nd key component of the model

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We assume that

$$\forall j \in V \setminus V_s, \qquad g_j = \frac{\sum_{i \in V_j^-} g_i x_{ij}}{\sum_{i \in V_j^-} x_{ij}} \qquad \textbf{(2)}$$

i.e., g of a blend of different gases is the weighted average of specific gravities of entering flows.

Project II: Balance of g in the system

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i.e., g of a blend of different gases is the weighted average of specific gravities of entering flows.

By multiplying **(2)** by the total entering flow:

$$g_j \sum_{i \in V_j^-} x_{ij} - \sum_{i \in V_j^-} g_i x_{ij} = 0, \forall j \in V \setminus V_s. \qquad \textbf{(3)}$$

Thus, **(3)** is adopted in the model.

Project II: Compute z *in the pipeline*

3rd key component of the model

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Literature suggests diverse methods to compute z

Experimental measurements, EoS methods (Dranchuk, 1975), empirical correlations (Katz et al., 1959) and regression analysis methods (Dranchuk, 1974; Gopal, 1977).

We have applied

The CNGA method:

$$z_{ij} = \frac{1}{1 + \frac{\overline{p_{ij}}\alpha 10^{\beta g_i}}{T^\delta}} \tag{4}$$

Assuming constant temperature

(4) can be written

$$z_{ij}(1 + \omega \overline{p_{ij}} \times 10^{\beta g_i}) = 1, \tag{5}$$

where $\omega = \frac{\alpha}{T^\delta}$ is an instance specific constant.

Project II: Compute z *in the pipeline*

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Thus, **(5)** is adopted in the model.

Project II: As a result

The following NLP model is proposed:

$$(\mathscr{M}_1) \quad Y_1 = \max \textstyle\sum_{i \in V_s} b_i \qquad (a)$$

s.t.:

Constraint defining g, z, the resistance of the pipeline and

$$\sum_{j \in V_i^+} x_{ij} - \sum_{j \in V_i^-} x_{ji} - b_i = 0, \qquad \forall i \in V \qquad (b)$$

$$3\overline{p}_{ij}(p_i + p_j) = 2(p_i^2 + p_j^2 + p_i p_j), \qquad \forall (i,j) \in A \qquad (c)$$

$$p_i^L \le p_i \le p_i^U, \qquad \forall i \in V \qquad (d)$$

$$b_i = 0, \qquad \forall i \in V \backslash V_s \backslash V_d \qquad (e)$$

$$b_i \ge 0, \qquad \forall i \in V_s \qquad (f)$$

$$b_i \le 0, \qquad \forall i \in V_d \qquad (g)$$

$$x_{ij} \ge 0, \qquad \forall (i,j) \in A. \qquad (h)$$

Constraints for g, z, **flow capacity estimates**, and **(c)** are non-convex, GO can become time consuming. Fast and possibly inexact solution methods may be required.

Project II: *Contributions*

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A more accurate ***NLP model*** *for*

Maximizing flow while considering the variability of g and z.

An extensive numerical experimentation:

- *GAMS* formulation for the NLP model.
  - Global optimizer: *BARON* (Tawarmalani & Sahinidis, 2004)
  - NLP local optimizer: *MINOS* (Murtaugh & Saunders, 1983)
  - *Heuristic algorithm* based on *an approximate model*

Through experiments, we have demonstrated that

- Neglecting $g-$ and $z-$ variation in instances where the variation is high, tends to give significantly misleading results.
- The proposed heuristic yields optimal or near-optimal solutions in most of the instances.

Project II: *Contributions*

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Project III: The line-packing problem

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Finally, let’s move to

PROJECT III

Project III: Problem statement

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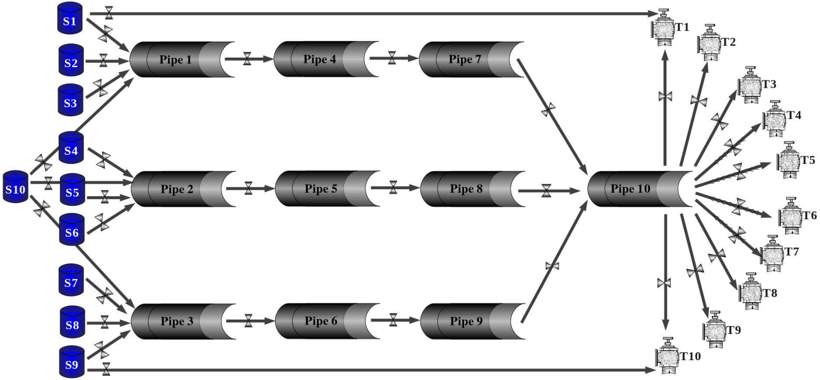
The optimization problem arises from one fact:

PIPELINES ARE

A means of gas transportation, BUT...

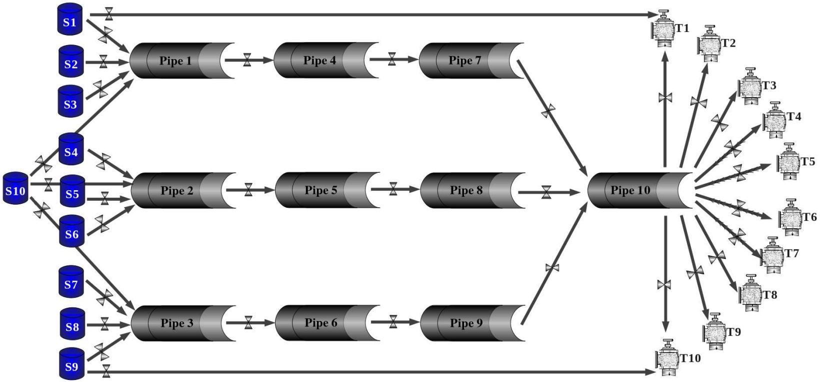


Project III: Problem statement

UiB

Pipelines also represent

Potential storage units for safety stocks!



Project III: Problem statement

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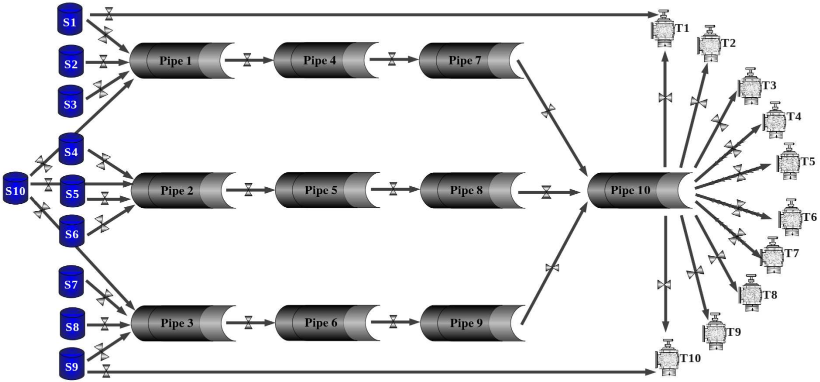
Hence, the *line-packing problem*

Optimization of gas storage in pipelines on a *short-term basis* to satisfy market demand.



Project III: Key statements

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Goal of the project

The efficient line-pack management as a strategy to meet market demand under scheduled events

Model:

Maximize total gas deliveries over a given planning horizon

We thus need to

1. **optimize** the refill of gas in pipelines in periods of sufficient capacity, and the withdrawals in periods of shortfall

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Project III: Main ideas behind the model

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Assumptions:

- Contracts specify upper bounds on, e.g., CO_2 content.
- CO_2 content is not equal at all sources.
- Flow into a pipeline = linear blend of entering flow streams
- Flow varies over time
- No blending inside the pipeline

Consequence:

A queue of batches with unequal CO_2 contents in the pipeline.

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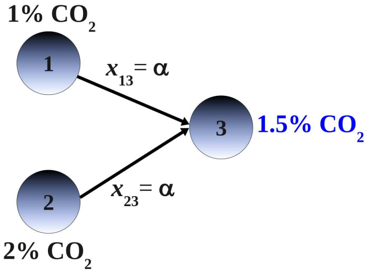

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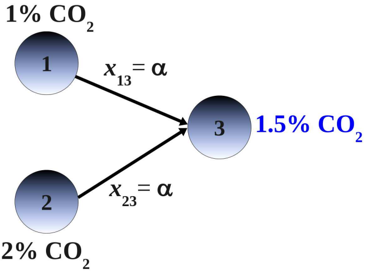

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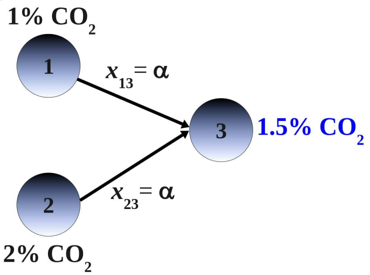

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Project III: Conclusions on the project

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Contributions:

A **MINLP model** for the *line-packing problem* by

Building up batches to meet market demand
over a given multi-period planning horizon

Observations from the numerical experiments:

- Test cases of moderate and small size were effectively handled by GO.

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Finally,

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CONCLUDING REMARKS

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After 3 wonderful years...

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  - 3 Papers published
  - 4 Presentations:
    - Sweden, France, Mexico & Norway

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And of course, **PhD thesis** → *Done!*

Thank you very much for listening...

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END.

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