Algorithms for deformable image registration

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Image registration

Image registration is the task of finding an optimal deformation field for alignment of an input image with a reference image.
Useful for comparison of

- Multimodal images (PET, CT, MRI, microscopy, histology
- Time series (microscopy, DCE-MRI)

Image registration

For example: Find average intensity value of 1 on the sky. Find the sky in modality 2.

Image registration - definitions

- Input image $f(x, t)$ (template)
- Target image $g(x)$ (reference image)
- Deformation field $u(x, t)$ (displacement field)
- Velocity $v(x, t)$
- Similarity measure D
- Regularizer R

Image registration - problem definition

Find the solution of

$$\min_{u} \int_{\Omega} D(x, u) + R(x, u) dx$$

where D can for instance be sum-of-squared differences

$$D = \frac{1}{2}(f(x + u, t) - g(x))^2$$

Methods for image registration

- Affine registration
  - Translation, rotation, scaling, shear
  - Parametric
- Deformable registration
  - All possible deformations u
  - Parametric or non-parametric
  - Diffusion
  - Spline
  - Curvature
  - Elastic*

Linear elasticity

- Linear elasticity is valid for small ∇u
- Minimization of total potential energy $V(u)$

$$V(u) = \int_\Omega \left(\underbrace{\frac{1}{2}\sigma : \epsilon}_{\text{strain energy}} - \underbrace{f \cdot u}_{\substack{\text{Work body} \\ \text{force } f}} \right) dx + \int_{\partial\Omega} \underbrace{t \cdot u}_{\substack{\text{Work surface} \\ \text{force } t}} \, ds \qquad (1)$$

$\sigma = 2\mu\epsilon + \lambda I tr(\epsilon)$, stress tensor
$\epsilon = \frac{1}{2}(\nabla u + \nabla^T u)$, strain tensor
f: volume force
t: surface force

Navier-Lame operator

- Variational operator δV leads to Navier-Lame operator

$$\underbrace{\mu \Delta u + (\mu + \lambda)\nabla(\nabla \cdot u)}_{\text{Minimization of strain energy } \sigma : \epsilon} + f = 0$$

- f is the functional derivative of the similarity measure
- In registration: f is unphysical
  - SSD
  - Cross correlation
  - Normalized gradients
  - Mutual information
  - ...

Cost function sum-of-squared differences (SSD)

- Valid for mono-modal images
- Expecting input and target image to have the same intensity in same location

$$D_{SSD} = \frac{1}{2}(f(x+u,t) - g(x))^2 \tag{2}$$

Cost function normalized gradients (NGF)

- Valid for multi-modal images with distinct edges
- Expecting input and target image to have the aligned or co-aligned gradient vectors

Given $f = f(x + u, t), g = g(x)$,

$$D_{NGF} = 1 - \left(\frac{\nabla f}{\sqrt{|\nabla f|^2 + \eta^2}} \cdot \frac{\nabla g}{\sqrt{|\nabla g|^2 + \eta^2}} \right)^2$$

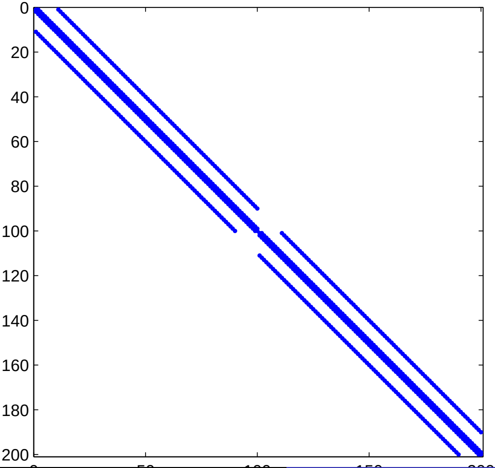
Numerical implementation of linear elasticity

- Discretize the Navier-Lame operator with central differences
- Obtaining a nonlinear set of equations

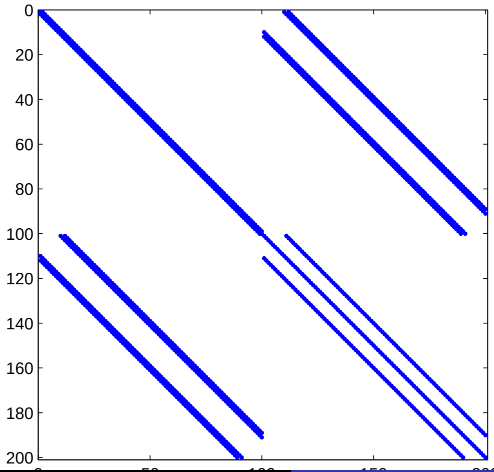
$$Au = (\mu A_1 + (\lambda + \mu) A_2) u = f(u)$$

- u is a long vector with voxel values (length $n \times 4$, n number of voxels)
- A is the discretized Navier-Lame operator
- A_1 is the discretized Laplace operator, $\Delta u = (\Delta u_1, \Delta u_1)$, $\frac{\partial^2 u}{\partial x^2} = \frac{u_{i+1,j,k} - 2u + u_{i-1,j,k}}{h_x^2}$
- A_2 is the discretized convective operator, $\nabla(\nabla \cdot u) = \nabla(\partial_x u_1 + \partial_y u_2)$

A_1 Laplace operator



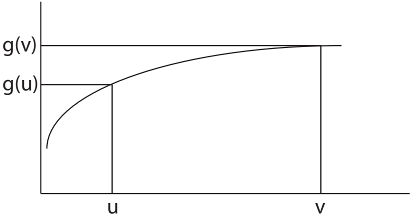
A_2 Convective operator



Fixed point iterations

- The system $u = A^{-1}f(u) = g(u)$ can be regarded as a fixed point iteration
- Will converge for a contractive mapping g

$$|g(u) - g(v)| \leq \lambda|u - v|$$



where $\lambda < 1$.

- Iterate until convergence

Multigrid

- Multigrid is an efficient solver for linear and nonlinear systems of equations
- Low frequencies on a fine level become high frequencies on coarse level
- Smoother: reducing the high-frequency errors by Jacobi or Gauss-Seidel
- Jacobi and Gauss-Seidel efficiently reduce the high frequency errors
- Restriction: Downsampling the residual error
- Prolongation: Interpolating a correction computed on a coarser grid into a finer grid

Multigrid

Generally, let

$$Au = f.$$

This is equivalent to

$$Au + Qu = f + Qu$$

leading to

$$Qu = (Q - A)u + f$$

Iterative scheme:

$$Qu^{k+1} = (Q - A)u^k + f$$

Multigrid

Note that

$$Qu^{k+1} = (Q - A)u^k + f$$

is equivalent to

$$u^{k+1} = u^k + Q^{-1} \underbrace{(f - Au^k)}_{\text{residual}} \tag{3}$$

Updating x with the error, and $Q^{-1} \approx A^{-1}$
Residual equation $Ae = r$.

Multigrid

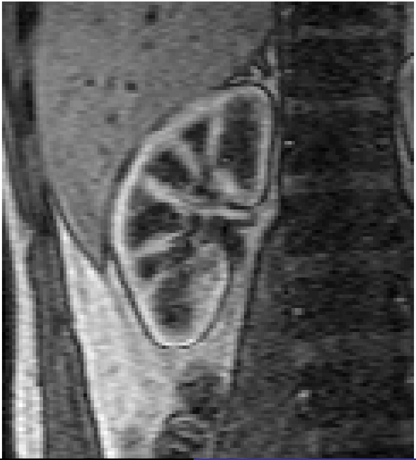
- Let $Q = \{a_{ii}\}$ (diagonal elements): Jacobi iterations
- $Du^{k+1} = (L + U)u^k + f$ where $A = D - L - U$
- Matrix free solver easily generated for each u_{ij} from the diagonal matrix D
- Guaranteed convergence if A diagonal dominant, $|a_{ii}| > \sum_{j,j\neq i} a_{ij}$

Multigrid

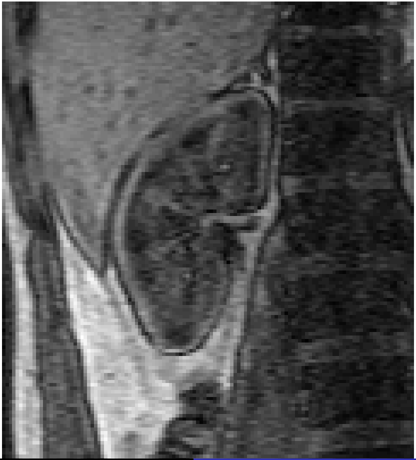
We treat the RHS as constant within a multigrid cycle, $Au = f(u)$.

- Relax ν_1 times on $A^h u^h = f^h$ with initial guess v^h
- Compute residual $r^h = f^h - A^h v^h$ and restrict to coarse grid $r^{2h} = I_h^{2h} r^h$
- Solve $A^{2h} e^{2h} = r^{2h}$ (Residual equation)
- Interpolate coarse-grid error to the fine grid by $e^h = I_{2h}^h e^{2h}$ and correct the fine-grid approximation $v^h = v^h + e^h$
- Relax ν_2 times on $A^h u^h = f^h$ with initial guess v^h

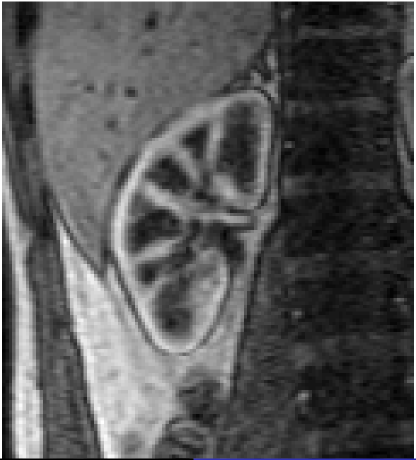
Input image f



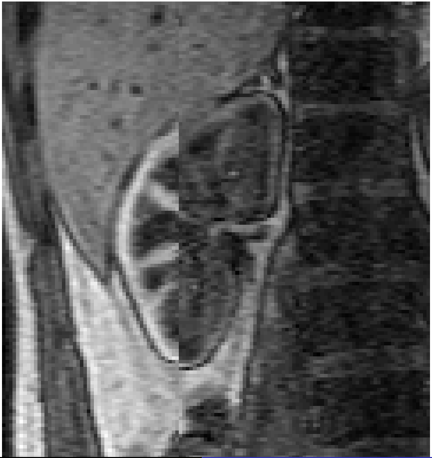
Target image g



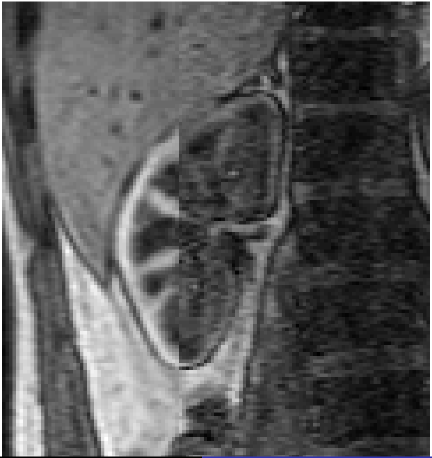
Registered image



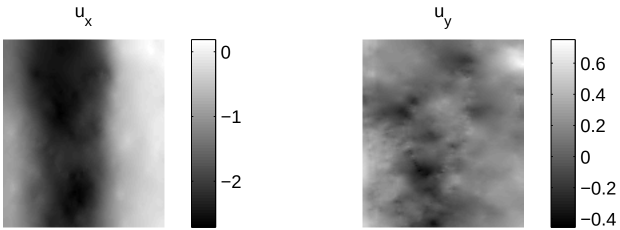
Checkerboards f,g



Checkerboards registered,g



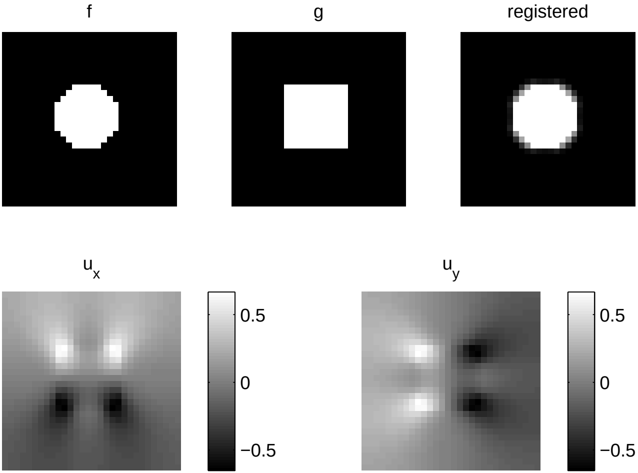
Deformation field $u = (u_x, u_y)$ (Unit: voxels)



What about larger deformations?

- Does linear elasticity support larger deformation gradients?

Registration of block image, linear elasticity



Fluid registration

- In *fluid registration* the deformation model is a highly viscous fluid deforming
- Fluid registration can therefore handle *large* deformations
- *Warning*: Can get singular deformations
- Monitor the Jacobian $J = |\nabla u|, du = JdU$

Fluid registration

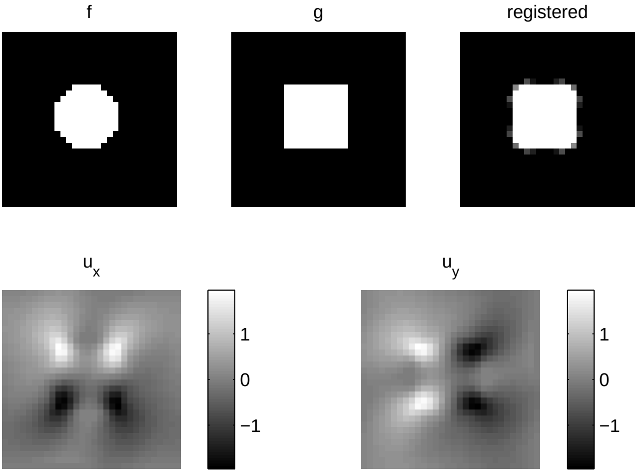
Navier-Lame operator (Navier-Stokes without acceleration and pressure term)

$$\mu \Delta v + (\mu + \lambda)\nabla(\nabla \cdot v) + f = 0$$

Deformation u is found from material derivative

$$v = \frac{\partial u}{\partial t} + v \cdot \nabla u = \frac{Du}{Dt}$$

Registration of block image, fluid registration



Evaluation of registration

How to evaluate registration?

- Visual inspection
- Cumulative inverse consistency error
- Control point evaluation

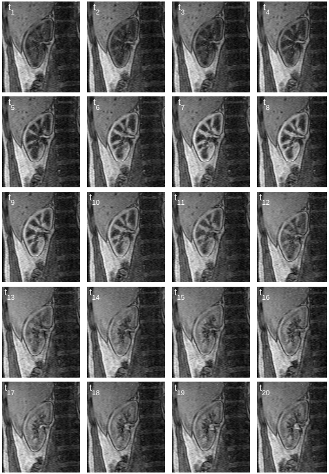
In particular for time series (DCE-MRI)

- Temporal variation
- Deviation to a compartment model
- Analysis of compartment model parameters
- Tools combining visualization with quantification???

DCE-MRI

- Bolus injection of contrast agent
- Gadolinium based
- Longer T1 times
- Measure the flow of the contrast
- Aim: Estimate glomerular filtration rate (GFR)

DCE-MRI time series



DCE-MRI time series

Sourbron model
Conservation of mass

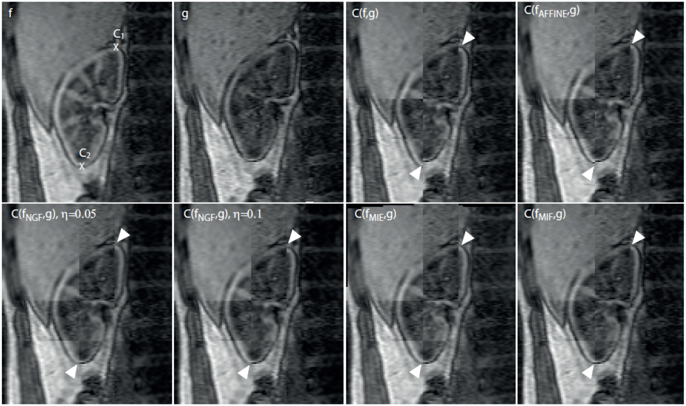
$$C = V_P C_P + V_T C_T$$

Rate of change of mass

$$V_T \frac{dC_T}{dt} = F_T C_T - (1 - f) F_T C_T$$

From Sourbton et al. 2008, Investi. Radiol.

Control point evaluation

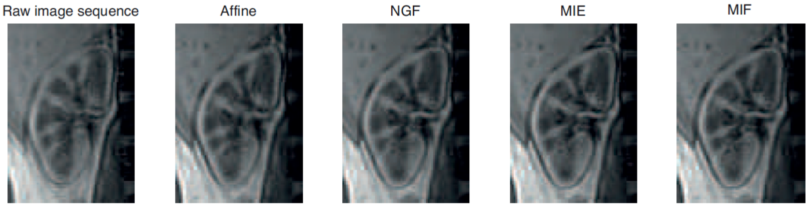


From Hodneland et. al, Normalized gradient fields and mutual information for motion correction of DCE-MRI time series, Proc. ISPA (2013)

Control point evaluation

Registration method	$\mu(C_1)$	$\mu(C_2)$	$\max(C_1)$	$\max(C_2)$
Unregistered	3.51	3.99	6.12	7.57
Affine	1.10	1.61	2.51	2.99
NGF	0.91*	0.74*	1.86*	2.17
MIE	1.42	0.87	3.00	2.11
MIF	1.10	1.11	2.61	1.75*

Blurryness



Temporal variation

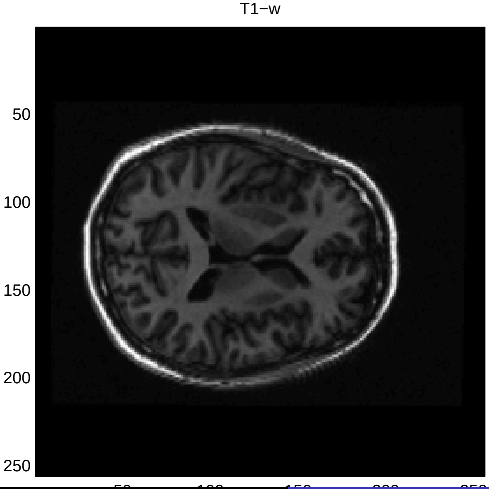
Subject/l,r	Unprocessed	Affine	NGF	MI
1/l	3.72	3.25	2.89	2.94
1/r	4.10	3.40	3.01	3.11
2/l	2.53	2.34	2.14	2.28
2/r	2.97	2.85	2.76	2.84
3/l	2.12	1.92	1.79	1.81
3/r	2.08	1.96	1.82	1.86
4/l	3.29	3.34	3.15	3.19
4/r	2.66	2.67	2.50	2.52
5/l	4.91	3.96	3.47	3.60
5/r	4.75	4.13	3.57	3.66
Average	3.31	2.98	2.71	2.78

From Hodneland et. al, Normalized gradient fields for motion correction of DCE-MRI time series, In revision, CMIG

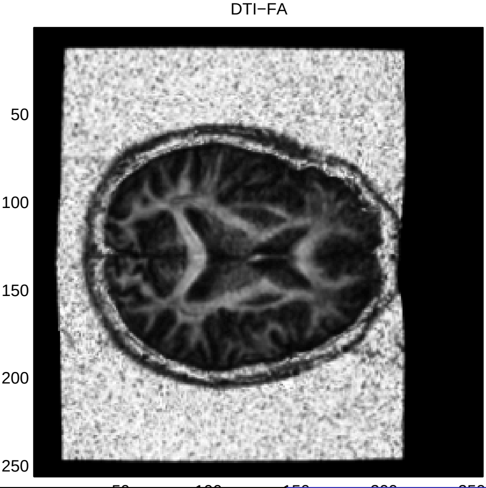
Evaluation of (DCE-MRI) time series

Evaluation of (DCE-MRI) time series

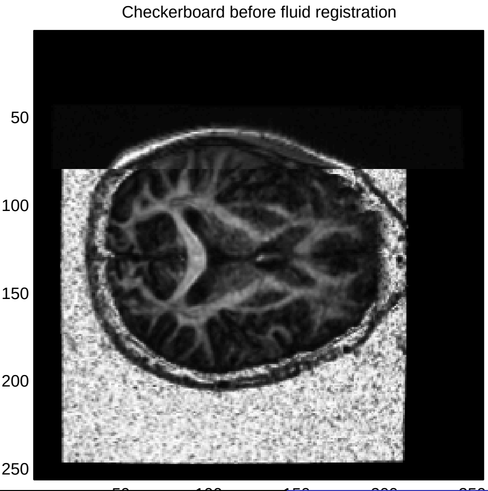
T1-w anatomy



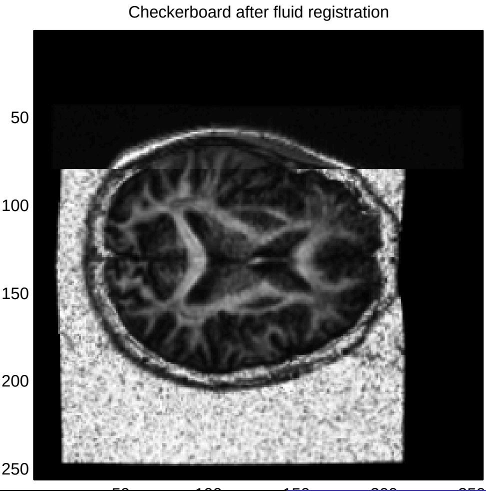
DTI-FA - distortions from Eddy currents



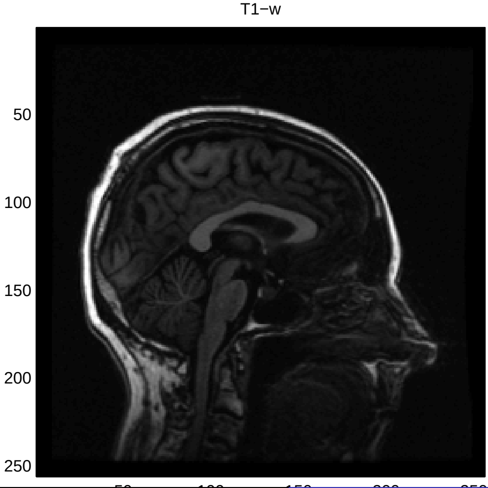
After affine registration



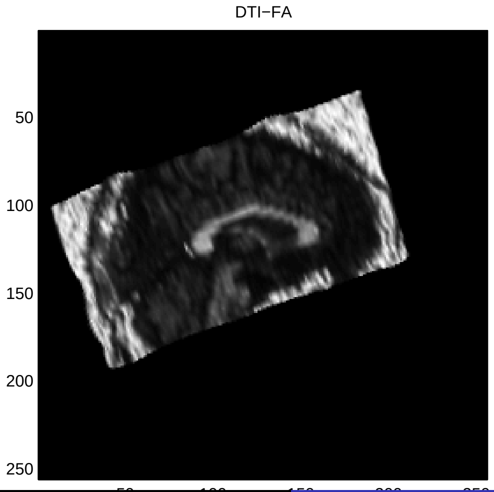
After fluid registration



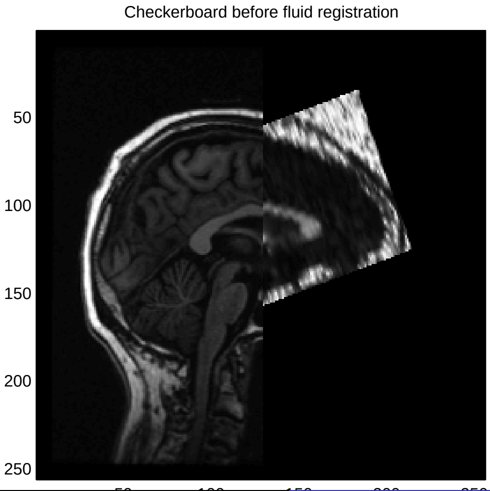
T1-w anatomy



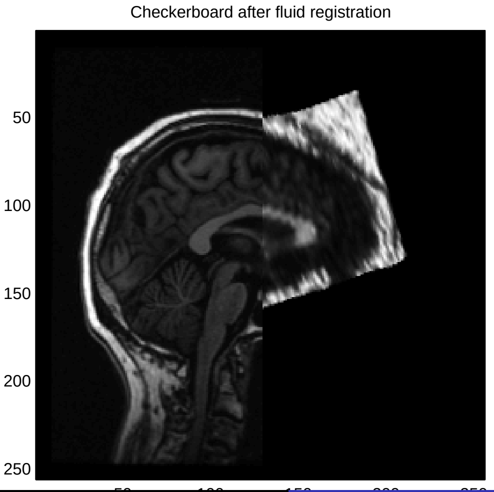
DTI-FA



After affine registration



After fluid registration



Conclusions

- Elastic image registration attempts to mimic the deformation of a deformable solid
- Fluid registration attempts to mimic the flow of a high viscosity fluid
- Image registration is a versatile tool using various models and similarity measures
- Challenges:
  - Large deformations
  - Missing structural information
  - Evaluation
- Still, image registration can account for many artifacts arising from motion or geometrical distortions